COMPARISON OF VEGETATION INDICES BASED ON SATELLITE-ACQUIRED SPECTRAL DATA

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ABSTRACT

Since the launching of Landsat I in 1972, investigators have derived numerous formulae for the reduction of multispectral scanner (MSS) measurements to a single value (vegetation index) for predicting and assessing vegetative characteristics such as plant leaf area, total biomass and general plant stress and vigor. This report summarizes the origin, motivation, and derivation of some four dozen vegetation indices. Empirical, graphical, and analytical techniques are used to investigate the relationships among the various indices. It is concluded that many vegetative indices are very similar, some being simple algebraic transforms of others.

1. INTRODUCTION

Current and accurate information on a global basis regarding the extent and condition of the world's major food and fiber crops is important in today's complex world. Traditional sampling techniques for estimating crop conditions, based on field collection of data, are time consuming, costly, and not generally applicable to foreign regions. An alternate approach is remote sensing - the science and art of obtaining information about an object, area, or phenomenon through the analysis of data acquired by a device that is not in contact with the object, area, or phenomenon under investigation [Lillesand and Kiefer (1979)].

A series of earth resources technology satellites (Landsats) have provided a way to monitor worldwide crop conditions since 1972. The sensor system onboard the Landsats, the multispectral scanner (MSS), measures the reflectance of the scene in four wavelength intervals (bands or channels) in the visible and near-infrared portions of the spectrum. The spectral measurements are influenced by the vegetation canopy, soil type, and atmospheric condition.

Investigators have developed techniques for qualitatively and quantitatively assessing the vegetative canopy from spectral measurements. The objective has been to reduce the four bands of Landsat spectral data to a single number for predicting or assessing such canopy characteristics as leaf area, biomass, percent ground cover, and plant population.

This report summarizes and references the origin, derivation, and motivation for some four dozen of these formulae which are referred to as vegetation indices (VIs). The VIs are categorized on the basis of statistical correlations and algebraic similarities. This analysis reveals the similarities of many vegetation indices.

2. LANDSAT DATA CHARACTERISTICS

Three Landsats have been launched since the summer of 1972, with Landsats 2 and 3 still operational. Each satellite is capable of providing 18-day repetitive coverage of the earth's surface. Each Landsat's onboard four-channel MSS system measures reflectance in four bands (fig. 1). The measurements are converted to digital counts and transmitted to receiving stations. Landsat MSS images cover an area of 185 by 185 kilometers and are composed of 7,581,600 picture elements (pixels) [Watkins and Freeden (1979)].

Typical reflectance patterns for herbaceous vegetation and soil are compared in figure 1. Dead or dormant vegetation has higher reflectance than living vegetation in the visible spectrum and lower reflectance in the near-infrared. Soil has higher reflectance than green vegetation and lower reflectance than dead vegetation in the visible, whereas in the near-infrared, soil has lower reflectance than green and dead vegetation [Tappan (1980)]. Jackson et al. (1980), Tucker and Miller (1977), and Deering et al. (1975) provide an extensive discussion of reflectance properties. Three papers of historical interest are Jordan (1969), Knipling (1970), and Pearson and Miller (1972).

3. DEVELOPMENT OF VEGETATION INDEX FORMULAE

Numerous vegetation indices have been used to make quantitative estimates of leaf area index, percent ground cover, plant height, biomass, plant population, and other parameters [Pearson and Miller (1972) and Wiegand et al. (1974)]. The formulae are based on ratios and linear combinations of the MSS bands.

The individual Landsat bands (CH4, CH5, CH6, CH7) have been used to estimate percent ground cover and vegetative biomass [Wiegand et al. (1974) and Seever et al. (1973)]. The correlation coefficients reported ranged from 0.295 for CH7 with crop over to 0.877 for CH6 with leaf area index. Similar correlations were reported by Tucker (1979).
Ratios of the Landsat bands have been used to estimate and monitor green biomass, etc. [Rouse et al. (1973, 1974), Carnegie et al. (1974), Johnson (1976), and Maxwell (1976)]. The obtained coefficients of determinations were slightly higher than those for the corresponding band differences. The twelve pairwise ratios (six of which are inverses of the other six) will be denoted by $R_5 = CH_4/CH_5$, $R_6 = CH_4/CH_6$, etc. Rouse et al. (1973, 1974) proposed using the normalized difference of Landsat channels 7 and 5 for monitoring vegetation, which will be referred to as $ND_7$. Deering et al. (1975) added 0.5 to $ND_7$ to avoid negative values and took the square root of the result in hopes of stabilizing the variance. This index is referred to as the transformed vegetation index and will be denoted by $TVI_7$. Similar formulae using channels 6 and 5 were proposed.

$$TVI_7 = \frac{0.5}{\sqrt{ABS(ND_7)}}$$

$$ND_6 = \frac{(CH_6 - CH_5)}{(CH_6 + CH_5)}$$

Our experience has been that the addition of 0.5 does not eliminate all negative values. We suggest the following computationally correct formulae:

$$TVI_6 = \frac{(ND_6 + 0.5)^2}{ABS(ND_6 + 0.5)}$$

$$TVI_7 = \frac{(ND_7 + 0.5)^2}{ABS(ND_7 + 0.5)}$$

where ABS denotes absolute value, and 0/0 is set equal 1. In section 6, it is shown that these formulae are equivalent for decision making to the basic ratios $R_5$ and $R_7$. Therefore, their use can only be justified if either they improve the regression fit or they normalize the regression errors [Draper and Smith (1966)].

Kauth and Thomas (1976) proposed an orthogonal transformation of the original Landsat data space to a new four-dimensional space. They christened this transformation the tassel cap transformation and named the four new axes soil brightness (SBI), green vegetation (GVI), yellow stuff (YVI), and non-such (NSI). The names attached to the new axes indicate the characteristics the indices were intended to measure.

$$SBI = 0.332 CH_4 + 0.603 CH_5 + 0.675 CH_6 + 0.262 CH_7$$

$$GVI = -0.283 CH_4 - 0.660 CH_5 + 0.577 CH_6 + 0.388 CH_7$$

$$YVI = 0.899 CH_4 + 0.428 CH_5 + 0.076 CH_6 - 0.041 CH_7$$

$$NSI = -0.016 CH_4 + 0.131 CH_5 - 0.452 CH_6 + 0.882 CH_7$$

Wheeler et al. (1976) and Miura et al. (1977) applied principal component analysis to Landsat data. The structure of the resulting transformation and the interpretation of the principal components are similar to those for the Kauth-Thomas transformation.

$$MSBI = 0.406 CH_4 + 0.600 CH_5 + 0.645 CH_6 + 0.243 CH_7$$

$$MGVI = -0.386 CH_4 - 0.530 CH_5 + 0.535 CH_6 + 0.522 CH_7$$

$$MYVI = 0.723 CH_4 + 0.597 CH_5 + 0.206 CH_6 - 0.278 CH_7$$

$$MNSI = 0.406 CH_4 - 0.039 CH_5 - 0.505 CH_6 + 0.762 CH_7$$

Miura et al. (1977) proposed another linear transform, based on the idea of spectral brightness and contrast. Generalizations of spectral brightness and contrast were defined in spectral density space, then transformed back to count space. The first two components of the resulting transformation are similar to the first two components of the preceding transformations.

$$SBSI = 0.437 CH_4 + 0.564 CH_5 + 0.661 CH_6 + 0.223 CH_7$$

$$SGVI = -0.437 CH_4 - 0.564 CH_5 + 0.661 CH_6 + 0.223 CH_7$$

$$SYVI = -0.437 CH_4 + 0.564 CH_5 - 0.661 CH_6 + 0.223 CH_7$$

$$SNSI = -0.437 CH_4 + 0.564 CH_5 + 0.661 CH_6 - 0.223 CH_7$$

Richardson and Wiegand (1977) used the perpendicular distance to the "soil line" as an indicator of plant development. The "soil line", a two-dimensional analogue of the Kauth-Thomas SBI, was estimated by linear regression. Two perpendicular vegetation indices were proposed.

$$PV_1 = \left(\frac{0.355 CH_7 - 0.149 CH_5}{0.355 CH_5 - 0.852 CH_7}\right)^{1/2}$$

$$PV_2 = \left(\frac{-0.498 - 0.457 CH_5 + 0.498 CH_6}{2.736 + 0.498 CH_5 - 0.543 CH_6}\right)^{1/2}$$

Evidently a minor error was made in the derivation of $PV_2$. The formula for $PV_2$ should be:

$$PV_2 = \left(\frac{-0.507 - 0.457 CH_5 + 0.498 CH_6}{2.736 + 0.498 CH_5 - 0.543 CH_6}\right)^{1/2}$$

These formulae are computationally inefficient and do not distinguish right from left of the "soil line" (water from green stuff). The standard formula from analytic geometry for the perpendicular distance from a point to a line solves this difficulty [Salas and Hille (1978)].

$$PV_1 = \left(\frac{1.091 CH_6 - CH_5 - 5.49}{1.091 CH_5 + 2.172 CH_6 + 0.431 CH_7}\right)^{1/2}$$

$$PV_2 = \left(\frac{2.4 CH_7 - CH_5 - 0.01}{2.4 CH_5 + 1.745 CH_7 - 0.431 R_45 - 1.35 R_47 R_45}\right)^{1/2}$$

The difference vegetation index (DVI) suggested by Richardson and Wiegand (1977) as computationally easier than PV_2, is essentially a rescaling of PV_1.

$$DVI = 2.4 CH_7 - CH_5$$

The Ashburn vegetation index [Ashburn (1979)] was suggested as a measure of green growing vegetation. The doubling of CH_7 is to make the scale compatible; CH_7 is 8-bit data and has one-half the range of the other three bands which are 6-bit data.

$$AVI = 2.0 CH_7 - CH_5$$

Colwell et al. (1979) proposed a vegetation indicator called greenness above bare soil (GRABS). This was another attempt to develop an indicator for which a threshold value could be specified for detecting green vegetation. The calculations were made using the Kauth-Thomas tassel cap transformation applied to sun-angle and haze-corrected data. The resulting index is quite similar to the GVI, since the contribution of SBI is less than 10 percent of GVI.

$$GRABS = GVI - 0.09178 SBI + 5.58959$$

Kanemasu et al. (1977) regressed winter wheat leaf area measurements on MSS band ratios and produced the following regression equation.

$$ELAI = 2.68 - 3.69 R_45 - 2.31 R_46 + 2.88 R_47 + 0.43 R_56 - 1.35 R_57 + 3.07 R_45 \times (-0.5 R_47) (R_45)$$

Pollack and Kanemasu (1979) later used a larger data set plus stepwise regression and obtained another regression equation.

$$CLAI = 0.366 - 2.265 R_45 - 0.431 (R_45 \times R_47) (R_45) + 1.745 R_45 + 0.057 PV_1$$
Separate regression equations were also obtained for CLAI values above and below 0.5.

\[
\text{LAI} = 1.903 - 1.138 \text{ R56} - 0.071(\text{R45} - \text{R47}) \text{R45} - 0.016 \text{PV16}, \text{ if CLAI is less than 0.5}
\]

\[
\text{LAI} = -5.33 + 0.36 \text{ PV17} + 0.54 \text{ TV16}, \text{ if CLAI is greater than 0.5}
\]

The Foreign Crop Condition Assessment Division (FCCAD) of the Foreign Agricultural Service (FAS), Houston, Texas uses another leaf area model. We have been unable to find any reference to the development of this model.

\[
\text{OLAI} = 41.325R45 - 42.45R46 - 1.903 - 1.138R56 - 0.071(R45 - R47)R45
\]

Badhwar (1981) proposed a ratio of GVI to SBI as a discriminator of crop discrimination. It will be shown in section 6 that this index is a generalization of a normalized difference.

\[
\text{GVS} = \frac{\text{GVI}}{\text{SBI}}
\]

Craig Wiegand suggested converting reflectance values to radiances. Linear transformations were used to change from reflectance to radiance values. Ratio and normalized difference formulae were also created using the radiance values.

\[
\text{RAD} = 0.0157 \text{ CH5}
\]

\[
= 0.0134 \text{ CH5} + 0.06
\]

\[
= 0.0139 \text{ CH5} + 0.03
\]

\[
\text{RAD7} = 0.0730 \text{ CH7}
\]

\[
= 0.0603 \text{ CH7} + 0.11
\]

\[
= 0.0603 \text{ CH7} + 0.03
\]

\[
\text{RAD75} = \frac{\text{RAD7}}{\text{RAD5}}
\]

\[
\text{NDBAD} = \frac{(\text{RAD7} - \text{RAD5})}{(\text{RAD7} + \text{RAD5})}
\]

Thompson and Wehassen proposed a technique utilizing transformed Landsat digital data to indicate when agricultural vegetation is undergoing moisture stress. The screening number or green number (GIN) was proposed to estimate the percentage of land in an area with a "healthy" cover of vegetation. A "soil line" is determined by inspecting the channel data and discarding data not considered reasonable for agricultural data. The "soil line" is then evaluated as the minimum value remaining in CH5 and subtracted from GVI to obtain GIN.

\[
\text{GIN} = \text{GVI} - \text{soil line}
\]

The data sets included in this study did not permit the computation of GIN. However, GIN is a linear transformation of GVI.

4. EVALUATION OF VEGETATION INDICES

4.1 Background

Richardson and Wiegand correlated eight VIs (GVI, DVI, SBI, PV16, PV17, TV16, TV17, and K57) with four plant component variables (crop cover, shadow cover, plant height, and leaf area index). The correlation coefficients obtained by plant component with the VIs (excluding SBI) were very similar. Later, Wiegand et al. (1979) correlated leaf area indices for winter wheat fields to five VIs (TV17, TV16, PV16, PV17, and GVI). The correlation coefficients by field and even between fields were similar.

Aaronson et al. (1979) studied the similarities and differences among seven VIs (AVI, DVI, GVI, OLAI, PV17, TV17, and KVI). The obtained correlation coefficients ranged from 0.8 to 1.0 and were stable from spring greenup to harvest. Aaronson and Davis (1979) later used a large data set, which included vegetation measurements and several VIs, to study interrelationships. The VIs (AVI, DVI, GVI, OLAI, KVI, PV16, PV17, TV16, and TV17) were correlated against each other and against vegetation measures such as plant height from tillering through harvest. The correlation coefficients between the VIs ranged from 0.8 to 1.0, and those between VIs and vegetation measures were similar.

4.2 Cluster Analysis of VIs

The similarity between the VIs was first studied using the NIDP program PIN, cluster analysis of variables. The absolute value of the bivariate correlations was used as the measure of distance between VIs, and the average distance between elements was used as the between cluster distance. Similar results were obtained using other standard distance measures.

This procedure separated the VIs into two large clusters plus a number of small clusters. One large cluster contained VIs based on MSS bands 5 and 7, which included AVI, PV17, TV17, and ND7. The other large cluster contained VIs, based on MSS bands 5 and 6, and a few VIs involving three or all four bands, which included GRB5, CLAI, ULAI, R65, TV16, ND6, GVI, MGVI, PV16, and GSVI. The VIs within these two clusters had absolute simple linear correlations greater than 0.90, with most greater than 0.95. The elements of these two large clusters are correlated at 0.8 or higher. Three smaller clusters readily apparent were: (NSI, R76), (R64, R76), and (GSI, NSBI, SSB1, SNEI). This clustering is applicable to the period from spring greenup to harvest. There are some clusters, however, which have high correlations for the whole season, especially those involving bands 5 and 7. The cluster trees on which this discussion is based are included in a more detailed report by Lautenschlager and Perry (1961).

Some VIs were not used in the cluster analysis because of their known relationships to others. The inverse ratios R54, R46, R47, R56, R67, and R57 were not used. DVI was discarded because of its relationship to PV17, as were RAD5, RAD7, RADR75, and NDBAD because of the linear relationships to CH5, CH7, R75, and ND7.

5. VEGETATION INDICES EQUIVALENCE

In this section, a definition of VI equivalence will be developed. This permits a natural categorization of the VIs. VIs are functions which associate a real value to the four-dimensional Landsat reflectance measurement vector, (MSS4, MSS5, MSS6, MSS7). Thus, it will be convenient to employ standard function notation: f:S1 → S2 denotes a function from the set S1 into the set S2; f(x), the value of f at the point (x) of S1; Dom(f), the domain of f; Ran(f), the range of f; and (f)−1, the inverse of f when it exists. The inverse exists if, and only if, f is one-to-one and onto. The composition of two functions has an inverse f, and only if, both functions have inverses; in which case

\[
(f \circ g)_{-1} = g_{-1} \circ f_{-1}
\]
Equivalence of VI's means their response surfaces determine precisely the same partition of the reflectance measurements space (equation 1). Elements of this partition are referred to as decision classes. Representative response surfaces and equivalence classes associated with TVI and R75 are illustrated in figures 2a and 2b. The nonlinear algebraic relationships exhibited above between R75, TVI and ND7 are illustrated graphically in figure 3.

Let and define the functions and by

\[ f(X5, X7) = \frac{(aX7 - bX5)}{aX7 + bX5} \]
\[ g(X5, X7) = \frac{X5}{X7} \]
\[ T(y) = \frac{b}{a} \left( \frac{1 + y}{1 - y} \right) \]

for and positive and less than one. Observe that is invertible; in fact

\[ T^{-1}(z) = \frac{(a - b)z}{(az + b)} \]

Thus, and are equivalent and the values of can be computed from the values of and vice versa.

Let and be real, and define the functions and by

\[ G(x) = (x - 1, 1) - (k - 1, k + 1) \] and \[ H(x) = (k - 1, k + 1) - (L, U) \] by

\[ G(v) = v + k \]
\[ H(w) = w[ABS(w)]^{p^{-1}} \]

for \( w \) between \( k - 1 \) and \( k + 1 \). \( L = (k - 1)[ABS(k - 1)]^{p^{-1}} \) and \( U = (k + 1)[ABS(k + 1)]^{p^{-1}} \), and \( u/0 \) defined as 1. It is easy to verify that and are one-to-one and onto and that

\[ (u \circ G \circ T^{-1} \circ g)(X5, X7) = \frac{(f(X5, X7) + k)[ABS(f(X5, X7) + k)]^{p^{-1}}}{f(X5, X7) + k} \]

Taking \( k = p = 1/2 \) and \( a = b = 1 \) show that the transformed vegetation index, TVI, is equivalent to the seven-five ratio, R75.

\[ (u \circ G \circ T^{-1}) \text{R75} = \text{TVI} \]

Figure 2. Response surface and equivalence classes.

Figure 3. TVI, ND7, and R75 versus time using data listed in Layenschlager and Perry (1981). All VI values have been rescaled 0 to 100.
Thus, the estimate, \(E_{\text{GVSB}}\), is equivalent to \(k65\) and \(ND6\). The relationships are illustrated graphically in Figure 4.

Using Landsat data, the following estimates were obtained [Lautenschlager and Perry (1981)].

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>CH4</td>
<td>6084</td>
<td>23.2</td>
</tr>
<tr>
<td>CH5</td>
<td>6084</td>
<td>26.7</td>
</tr>
<tr>
<td>CH6</td>
<td>6084</td>
<td>41.4</td>
</tr>
<tr>
<td>CH7</td>
<td>6084</td>
<td>17.5</td>
</tr>
</tbody>
</table>

From these estimates, one easily obtains the regression equations

\[
\begin{align*}
\text{CH7} &= 0.4100 \times \text{CH6} - 0.5100 \\
\text{CH4} &= 0.9236 \times \text{CH5} + 6.564
\end{align*}
\]

Naively substituting into the formula for CVI gives the following.

\[
\begin{align*}
\text{GVI} &= 0.74 \times (\text{CH6} - 1.14 \times \text{CH5} + 0.3) \\
\text{SB1} &= 0.78 \times (\text{CH6} + 1.03 \times \text{CH5} + 2.96)
\end{align*}
\]

Using the information in the above tables pertaining to the expected range of the data, it is easy to see that a rough approximation for \(E_{\text{GVSB}}\) is:

\[
E_{\text{GVSB}} = \frac{(\text{CH6} - 1.14 \times \text{CH5})}{\text{CH6} + 1.03 \times \text{CH5}}
\]

which is approximately \(ND6\). In fact, let

\[
\begin{align*}
\text{h}(v) &= (a + v d)/(a - v c) \\
\text{k}(x, y) &= (a x - b y)/(c x + d y) \\
\text{r}(x, y) &= x/y, \text{then} \ h(k(x, y)) = x/y = r(x, y)
\end{align*}
\]

Thus, the estimate, \(E_{\text{GVSB}}\), is equivalent to \(k65\) and \(ND6\). These relationships are illustrated graphically in Figure 4.

<table>
<thead>
<tr>
<th>Variable</th>
<th>CH4</th>
<th>CH5</th>
<th>CH6</th>
<th>CH7</th>
</tr>
</thead>
<tbody>
<tr>
<td>CH4</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CH5</td>
<td>0.86</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CH6</td>
<td>0.73</td>
<td>0.64</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>CH7</td>
<td>0.67</td>
<td>0.50</td>
<td>0.96</td>
<td>1.00</td>
</tr>
</tbody>
</table>

6. SUMMARY AND CONCLUSIONS

Other researchers have studied the relationships among a few of the VIS considered in this report. Past work has been based exclusively on correlation analysis. Aaronson and Davis (1979) showed conclusively that, during the spring greenup to harvest phase of the crop season, the VIS used operationally by The Foreign Agriculture Service (FAS)/Foreign Crop Condition Assessment Division (FCCAD) were highly correlated and had similar correlations with various plant components such as biomass, plant height, etc.

This study extends analysis to include all VIS found in the literature. Techniques used to investigate relationships between the VIS included variable clustering by correlation, graphical presentations, and functional equivalence for decision making. Variable clustering separated out two large clusters of VIS. One cluster contained those VIS which used channels 5 and 7 data. The other cluster contained VIS using channels 5 and 6 data plus some VIS using all four channels of data. The variable clustering technique also showed that these two clusters were highly correlated. The relationships were stable during the spring greenup to harvest period of the crop season. Graphical presentations reinforced the clustering results, illustrating the relationships over time and through response surfaces. Mathematical techniques were used to formalize the idea of VIS equivalence. This equivalence was used to confirm relationships observed earlier and to investigate less apparent relationships.

LITERATURE CITED


Aaronson, A. C.; L. L. Davis; and G. A. Hey, 1979, "Results of the Vegetative Index Correction Study", Tech. Memo. No. 6, Crop Condition Assessment Division, USDA/FAS, Houston, TX.


DISCUSSION

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#1. Kalsbeek, Mendoza, and Eudescu: New Model

The proposed new model is an expression that gives travel expenses as a function of size of
interviewer assignment area. No. of PSU's in area, No. of PSU's to be visited in one trip and No. of
callbacks to be made. It extends work of H&M
and is most welcome. The derivation seems emi-
nently reasonable and actual survey expenditures
should be found to follow the functional form,
even after further experiences or a review of exist-
ing ones would be required to establish the con-
formity between actual expenditures and the
expression.

The authors compare recommended optimum PSU
sizes based on three expressions for travel expen-
ses: the simple one, the H&M and the new one.
It seems that the recommendation based on ignoring
travel expenses, the simple one, calls for too
small PSU's, although there is no very serious
loss of precision until the survey is taken to
cover all of the US. In practice one may prefer
to use the optimizing formula based on the simple
model but also use some judgment in changing
C_8^2 and C_8^2 so as to take account of travel
expenses. This judgment could be sharpened by
applying the authors' vision of how interviewers
travel about.

#2. L. R. Ernst: Controlled Selection

The paper furnishes a way of tightening the
control of the two-way stratification method
given by Bryant, Hartley and Jessen (1960).
I have been calling their method "merging random
permutations" because of the way I carry it out.
That is, a two-way stratification design selection
can be exhibited as two columns of strata identi-
fiers one for each "way." The two identifiers in
each row point to a cell where a selection is to
be made. By permuting the second column the cell
selections are changed but the marginal selection
numbers are "controlled." If there are, for
example, two or more identifiers for strata in
both ways then cells may be visited once, or two
times and this may constitute too much loss of
"control."

The author's method, if a solution exists,
allows cells to be hit zero or one times, or one
or two times, or two or three times, etc.,
without more flexibility relative to cell quotas.
This method may be called "deep control," in
parallel to the terminology "deep stratification"
that describes multi-way subdivision of the popu-
lation. I wonder if my merging random permuta-
tions approach could not be used after satisfying
cell quotas up to none or one additional selection.

#3. Drummond: Workload Bias

The paper describes a variety of options for
scheduling field work with a sympathetic apprecia-
tion for the realities of enumerating. The title of
the paper suggests that imposed randomization
might combat bias. Although I found expressions
for inclusion probabilities, I don't believe there
was even an expression for the entire, much
less its bias or variance. Since there is some
cost to randomize, if only the looking at a random
number, there ought to be some reduction in bias,
strata although ground-based measurements are available from some but not all strata. These extensions also direct our attention more to the proportions of various kinds of misclassifications as well as to a summary measure of agreement.

#6. Lautenschlager and Perry: Comparison of Vegetation Indices

The paper furnishes background information on remote sensing using the Landsat bands that I found most fascinating. The listing of indices was less gripping, but their clustering was of some reasonableness. Then the authors introduce the concepts of decision rules and equivalence classes that seem very close to the notions of test in statistical inference. I began to look for a comparison of indices in terms of, say, their asymptotic relative efficiencies, but couldn't find it. Actually the paper seemed to stop in mid-argument. It was marked "Working Draft" and perhaps the final version will arrive at some comparison of power or of efficiency.