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Response Errors in the Weighted Estimator

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ABSTRACT

Farm operators selected from the 1981 June Enumerative Survey (JES) in three states were reinterviewed to measure the repeatability of responses for "farm acres operated." This farm characteristic is used to obtain weighted estimates for livestock items and the number of farms. Only 31 percent of the operators reported the same value for farm acres operated on both interviews. Also, about one-third of the responses during the reinterview were not even within 10 percent of the JES value. Despite the response differences, most of the weighted estimates from the two interviews were not significantly different. The extent to which the response errors decreased the precision of the estimates is not known.

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SUMMARY

A reinterview study was conducted in three states after the 1981 June Enumerative Survey (JES) to measure the repeatability of responses for the survey item "number of farm acres operated." The responses for this item from the JES are used to derive the weights for weighted estimates of numerous livestock items and the number of farms. Therefore, the quality of many estimates is dependent upon the number of farm acres operated being accurately reported.

The responses for farm acres operated often differed between the JES and reinterview study. The same value was reported on both occasions by only 31 percent of the operations in the three states. About one-third of the reinterview responses were not even within 10 percent of the JES value. The responses were inconsistent even when the respondent or enumerator was the same for each interview. The component of farm acres operated with the most inconsistent responses was land rented from others.

The weighted estimates for hog and cattle inventories were not significantly different between interviews due to offsetting reporting differences in the number of farm acres operated. The weighted estimate of the number of farms was significantly different between interviews in only one of the three states. Therefore, although farmers generally did not report the same number of farm acres operated on each interview, the differences in the responses did not result in significant biases in the weighted estimates.

The reinterview study was not designed to measure the effect of the response differences on the coefficient of variation (C.V.) for each estimate. Therefore, the extent to which the C.V.'s were increased as a result of obtaining different answers from a respondent on each occasion is not known.

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INTRODUCTION

The Statistical Reporting Service (SRS) used a weighted estimator in 29 states during 1981 to obtain multiple frame livestock estimates. In addition, weighted estimates for livestock items and the number of farms were calculated in 16 of these states based solely on the area frame. The weight or proration factor in each weighted estimate is the ratio of tract acres operated to the farm acres operated. This weight is obtained from each farm operation which operates land in a segment sampled from the area frame.

The Survey Research Section conducted two reinterview studies during the 1970's to evaluate the quality of the data reported for this weight. These studies concentrated on the denominator of the weight--farm acres operated--which was considered much more susceptible to nonsampling errors than the tract acres operated.

The first study took place in Nebraska immediately following the 1974 June Enumerative Survey (JES). The results indicated that farm acres operated was not being accurately reported. ^{2/} However, due to offsetting reporting errors, no bias was uncovered in the reporting of farm acres.

The second study, which was conducted in conjunction with the 1977 December Enumerative Survey (DES) in Indiana, North Carolina and Oklahoma, revealed that the farm acreages reported during the DES were biased downward in each state. ^{6/} The primary cause of this bias was the failure to include land not actively in use such as woodland, wasteland and idelnd. A downward bias in the farm acres results in an upward bias in the weighted estimates. Therefore, the study concluded that entire farm acres must be more accurately reported if the current weighting method was to continue in these states.

More recently, analysis of the "frozen weights" procedure of SRS during the March 1981 Multiple Frame Hog Survey again showed the difficulty of obtaining correct data for farm acres operated. ^{3/} On 11.5 percent of the interviews, the respondent claimed that the farm acreage from the December survey, which was recorded on the March questionnaire, was incorrect.

This report discusses a reinterview study carried out after the 1981 JES to determine if problems still exist with the reporting of farm acres operated. This paper primarily reports on the consistency of the responses for "farm acres operated" between the JES and followup interviews and evaluates the effect of reporting differences on the weighted estimates.

SAMPLE DESIGN

A sample of farm operations from the 1981 Objective Yield (OY) Surveys in Minnesota, North Carolina and Ohio was used in the reinterview study. Minnesota, North Carolina and Ohio were selected because related research on weighted estimates was being conducted in these states. 8/ The sample was selected from the OY in order to reduce data collection costs since the reinterviews could be completed during the initial OY interview. The JES responses for items such as farm acres operated were available for each operation to be reinterviewed since the OY sample is selected from the JES sample.

The reinterview sample included the OY Surveys for wheat, corn and soybeans in Minnesota, corn in North Carolina, and corn and soybeans in Ohio. To reduce data collection costs, the reinterview study only involved farm operations that were randomly selected to be contacted around July 1 or August 1 for the OY Surveys. All of the reinterviews in North Carolina and half of the reinterviews from the wheat OY in Minnesota were conducted during the July 1 time period. The remaining reinterviews were carried out during the August 1 OY initial interviews. Only those operations that responded during the JES to the questions being asked in the reinterview were included in the followup interview.

The reinterview sample is not a sample from the population of all farm operations in each state. The inference level for each state is restricted to a particular subset of farm operations. In Minnesota, the population includes farm operations having wheat, corn or soybeans that would have responded to the JES. The population for North Carolina includes only corn growers while the Ohio population involves corn and soybean operations that would have responded to the JES. Therefore, inferences will pertain only to these types of operations and not all operations.

Table 1 summarizes the information concerning the reinterview sample for each state and the three states combined. Shown in this table are the number of reinterviews attempted, the number of reinterviews completed, the average number of farm acres operated for the reinterview sample based on the JES responses, and the Crop Reporting Board's official average farm size for 1981. Notice that the average farm size for the reinterview sample in each state was considerably higher than the official average that was based on all farm operations that sold or normally sell at least \$1,000 of agricultural products. These acreage differences result from the fact that the reinterview study was based only on a particular group of farm operations, which were quite different in size from the average farm.

Table 1--The number of reinterviews attempted and completed, the average number of farm acres operated for the reinterview sample based on the JES responses, and the official average farm size in 1981.

State	Number of Reinterviews Attempted	Number of Reinterviews Completed	Average Number of Farm Acres Operated for the Reinterview Sample	Average Number of Farm Acres For All Farms
Minnesota	185	171	525	291
North Carolina	115	113	278	125
Ohio	139	130	457	173
Three States Combined	439	414	438	201

REINTERVIEW APPROACH

The reinterview format for this study was different from the two reinterview projects conducted during the 1970's. The approach in the earlier projects was to determine the "true" farm acres operated, to the extent possible, so that inferences about reporting bias could be stated. To accomplish this objective, a more detailed and probing questionnaire was designed for the reinterview than the original interview.

In order to reduce data collection costs and respondent burden, the 1981 reinterview study did not use a more detailed questionnaire. The same questions were asked to obtain farm acres operated during the reinterview that were asked during the JES. No attempt was made to determine the "true" farm acres for this study. The major objective of the reinterview was to measure the repeatability of the responses between the two interviews. Therefore, inferences will be made concerning response inconsistency rather than response bias. The section of the JES questionnaire pertaining to the farm acres operated is shown in Appendix A. The reinterview questions are identical so are not presented.

Changes in the size of the operation since the JES interview were also obtained during the reinterview. The responses were adjusted to reflect these changes so that comparisons of the data between the two interviews would exclude actual acreage changes. Also, responses that were edited during the JES or reinterview were changed to the responses originally recorded on the questionnaire so that response differences could not be attributed to office editing.

REPEATABILITY OF RESPONSES

The estimates presented in this report are weighted averages that take into consideration the expansion factor for each operation. The expansion factor is the inverse of the probability that the operation would be selected for the OY Survey. To account for nonresponse in the reinterview sample, the expansion factors were adjusted independently within each JES land use stratum that had nonrespondents during the reinterview.

Because of the complex survey design, each segment in the reinterview was randomly assigned to one of ten replicates to simplify the statistical testing. This approach has been used in previous research work by the Survey Research Section. 4/7/ A brief explanation of the replication method is given in Appendix B. To protect against the effect of the random assignment, the assignment process was done three separate times and the testing conducted on each assignment. Inferences were based on the average of the three tests. An average significance level less than or equal to .05 was considered significant throughout this report.

The responses for "farm acres operated" were compared between the two interviews for each operation. Table 2 summarizes the results of the comparisons for each state and the three states combined. Surprisingly, a very small percentage of the operators reported the same value on both interviews. These percentages are smaller than the percentages from the two earlier reinterview studies.

The same value was reported in each interview by 68 percent of the operators during the 1974 Nebraska study and 56 percent of the operators during the three-state study in 1977. The percentages may be higher from the earlier studies because of their reinterview approach. In these studies, the farm acres operated was obtained during the reinterview and then the operator was told the acreage given in the original interview. The operator then reconciled the differences. Therefore, the operator was allowed to change the reinterview acreage to that reported on the original interview. The operator was not provided the reported acreage from the JES during the 1981 reinterview study so the opportunity did not exist to change the reinterview response to the JES response.

Not only did a large percentage of operators report a different value on the reinterview, but in many cases the difference was not even within 10 percent of the JES value. Overall, about one-third of the operations reported a value for farm acres operated on the reinterview that was not within 10 percent of the JES response.

Table 2--The percentage of operators who reported a value on the reinterview for farm acres operated which was the same, within 5 percent, within 10 percent, and within 20 percent of the response during the 1981 JES.

State	The Number of Farm Acres Operated			
	Same Value	Within 5%	Within 10%	Within 20%
Minnesota	40.4	68.5	77.6	82.5
North Carolina	17.6	41.6	52.6	78.1
Ohio	29.7	54.1	64.5	83.7
Three States Combined	30.7	56.3	66.4	81.9

Exploratory analysis was performed to determine if any reasons for the response discrepancies could be discovered from the available data. The analysis first focused on whether the inconsistencies might be the result of:

- (1) interviewing a different person on the two occasions or
- (2) conducting the interview with a different enumerator on the reinterview.

The respondent was coded on the questionnaires as being the same in both interviews for 90.1 percent of the interviews in Minnesota, 83.5 percent in North Carolina and 84.6 percent in Ohio. The same enumerator handled both interviews on 63.7 percent of the interviews in Minnesota, 93.9 percent in North Carolina and 44.6 percent in Ohio.

Table 3 provides the same summary information as Table 2 except that the analysis is based only on situations where either the respondent or the enumerator was the same for both interviews. Notice that the proportion reporting the same or a similar value on the reinterview was still small. Therefore, the responses were inconsistent even when the same respondent or enumerator conducted each interview.

The value for farm acres operated was derived during each interview by separately obtaining three components--land owned, land rented from others and land rented to others. The farm acres operated is the sum of the land owned and rented from others minus the land rented to others. The responses for each of the three components were analyzed to determine if the response differences for farm acres operated were caused mainly by a particular component.

Table 3--The percentage of operators who reported a value on the reinterview for farm acres operated which was the same, within 5 percent, within 10 percent, and within 20 percent of the response during the 1981 JES when either the respondent or enumerator was the same for both interviews.

State	The Number of Farm Acres Operated							
	Same Respondent				Same Enumerator			
	Same Value	Within 5%	Within 10%	Within 20%	Same Value	Within 5%	Within 10%	Within 20%
Minnesota	42.7	69.4	78.5	82.8	41.3	73.0	81.2	85.1
North Carolina	17.2	43.3	55.0	81.5	17.8	41.9	53.2	79.0
Ohio	27.2	52.6	63.9	86.7	38.8	66.0	72.7	95.5
Three States Combined	30.8	56.9	67.5	83.9	32.1	59.8	68.6	86.0

Table 4 contains the average farm acres operated and average acreages of each component for each state and the three states combined using the JES responses from the reinterview sample. These averages are based only on individual and partnership operations since acreages for the three components are not asked of managed operations. In each state very little land was rented to other people, but a large portion of the land operated was rented from others.

Table 4--The average number of farm acres operated, acres owned, acres rented from others and acres rented to others for individual and partnership operations in the reinterview study based on their JES responses.

State	Average Number of Farm Acres Operated	Average Number of Acres Owned	Average Number of Acres Rented from Others	Average Number of Acres Rented to Others
Minnesota	525	277	262	14
North Carolina	270	115	157	2
Ohio	451	192	265	6
Three States Combined	434	205	237	8

The percentage of operators reporting the same value for each component of land on both interviews is given in Table 5. Also shown in this table is the percentage of operators reporting a value on the reinterview within 10 percent of the JES value. The percentages were low in each state for land owned and land rented from others. The percentages for land rented from others were always lower than the percentages for land owned, and in some instances were significantly lower. The percentage reporting the same value was significantly lower for land rented from others in Ohio and the three states combined while the percentage reporting a value within 10 percent of the JES response was significantly smaller in North Carolina, Ohio and the three states combined. Therefore, although reporting problems existed with land owned and land rented from others, the problems were more serious for land rented from others.

The consistency of responses for farm acres operated and its components was always lowest in North Carolina. Based on the JES responses for the reinterview sample, almost one-fourth of the farm acres in North Carolina was classified as waste, woods, roads and ditches compared to only 7.3 percent in Minnesota and 5.0 percent in Ohio. The high rate of response inconsistency in North Carolina tends to support the findings from Hill and Farrar ^{6/} that the primary cause of inaccuracies in reporting farm acreage is the reporting of woodland, wasteland and idledland.

Table 5--The percentage of operators reporting the same value on both interveiwis for each component of farm acres operated or a value on the reinterview that was a within 10 percent of the 1981 JES response.

State	Land Owned		Land Rented from Others		Land Rented to Others	
	Same Value	Within 10%	Same Value	Within 10%	Same Value	Within 10%
Minnesota	63.5	81.2	54.2	75.5	93.4	94.5
North Carolina	46.5	72.0	33.2	49.0	87.9	88.4
Ohio	55.7	74.4	42.2	65.4	93.4	96.4
Three States Combined	56.4	76.3	44.5	65.2	92.1	93.7

The average acreage difference (reinterview minus JES) for farm acres operated was 13.3 in Minnesota, 14.3 in North Carolina and -2.9 in Ohio. None of these differences were significantly different from zero. That is, neither interview resulted in a significantly higher or lower average farm size. This was anticipated since the same questionnaire format was used on each interview. Despite the lack of significance in the acreages, the weighted estimates for livestock items and the number of farms might be significantly different between interviews. This topic will be addressed in the next section.

Finally, the average of the absolute value of each acreage difference for farm acres operated was 39.4 in Minnesota, 51.0 in North Carolina and 42.6 in Ohio. Each of these absolute differences was significantly greater than zero.

In summary, the responses for "farm acres operated" were very inconsistent between the JES and followup interviews in each of the three states tested. Response discrepancies occurred frequently even when the respondent or enumerator was the same for both interviews. The responses were inconsistent regardless of whether the land was owned or rented from others. However, the percentage reporting the same or a similar response in both interviews was always smaller for land rented from others and in some cases was significantly smaller.

EFFECT ON THE WEIGHTED ESTIMATES

The weighted estimates were compared between the JES and reinterview using the reinterview sample of 414 operations. These estimates were compared to determine if the response differences for farm acres operated significantly affected the estimates. The tract acres operated and the item being estimated such as total hogs and pigs, which were obtained from the JES, were used in the weighted estimates for the JES and reinterview. Therefore, differences in the weighted estimates only reflect response differences for farm acres operated.

Weighted estimates were computed for three variables from the JES--number of farms, number of hogs and pigs, and number of cattle and calves. All operations in the reinterview sample were classified as farms during the JES. Therefore, the comparison of the weighted number of farms estimates is also a comparison of the weights. Only 26.0 percent of the operations had any hogs and 44.2 percent had cattle. Therefore, only the acreage responses for operations with hogs or with cattle will affect the estimates pertaining to livestock.

Table 6 gives the relative difference between the reinterview and JES weighted estimates for each state and the three states combined. In only one instance was the weighted estimate significantly different for the two interviews. This occurred in Minnesota for the number of farms estimate.

Table 6--The relative difference between the reinterview and JES weighted estimates. 1/ The symbol, *, denotes significantly different from zero.

State	Relative Difference (%)		
	Number of Farms	Hogs and Pigs	Cattle and Calves
Minnesota	-11.2*	-0.1	-2.0
North Carolina	-4.7	-0.8	0.8
Ohio	1.5	1.7	2.1
Three States Combined	-5.6	0.2	-0.8

1/ Relative Difference = (Reinterview - JES)/JES.

TOTAL VARIANCE

In most surveys, the assumption is made that if the survey were to be repeated with the same sample that the responses for an item would vary little, if any, for each respondent from one interview to the next. Under this assumption, the sampling variance is used as the measure of precision for an estimate. However, if the answers from respondents differ from interview to interview, the sampling variance makes the estimates appear more precise than they actually are since the response variability from interview to interview is not taken into account. When response errors exist, the appropriate measure of precision is the total variance rather than the sampling variance.

Hansen, Hurwitz and Bershad ^{5/} have shown that the total variance for a survey item is given by:

$$\text{Total Variance} = \text{Sampling Variance} + \text{Response Variance} + \text{Covariance of Response and Sampling Deviations.}$$

The response variance is the sum of two components--the simple response variance and the correlated component of the response variance. The simple response variance is the variance of the individual response deviations over all possible interviews. The correlated component of the response variance is the variance in responses caused by factors such as interviewers and supervisors. The simple response variance is usually much smaller in value than the correlated component. The covariance of response and sampling deviations is zero when the survey is repeated for a fixed sample of units as was done in this reinterview study. In other situations, the covariance is generally assumed to be negligible. Therefore, the major elements of the total variance are usually the sampling variance and the correlated component of the response variance.

The 1981 reinterview study was not designed to provide an estimate of the total response variance for farm acres operated. To estimate the total response variance, the enumerator for each reinterview would have to be different from the enumerator from the JES. This was not feasible since this study was conducted in conjunction with the OY initial interviews.

An estimate of the simple response variance, which is usually only a small part of the total response variance, was calculated for three weighted estimates -- number of farms, total hogs and pigs and total cattle and calves. The simple response variance for each estimate only took into account the response deviations between interviews for "farm acres operated" and not the response differences for other components of the weighted estimate such as the tract acres operated and the total hogs and pigs. The effect of the simple response variance on the precision of each of the three estimates was small. For example, the coefficients of variation at the three state level based solely on the sampling variance were 6.12, 30.05 and 12.58 percent, respectively, for the estimates of number of farms, hogs and cattle. The coefficients of variation for these estimates based on the sampling and simple response variances were 6.66, 30.05 and 12.62 percent.

A study was conducted in selected Iowa counties during 1970 that measured the total response variance from farm operators for 21 farm items. ^{1/} When averaged over the 21 farm items, the response variance accounted for about 20 percent of the total variance. Looking at some of the individual farm items, the response variance accounted for 1.5 percent of the total variance for farm acreage, 5.1 percent for land rented from others, 44.5 percent for idleland, 2.1 percent for cattle and calves, 20.2 percent for breeding hogs, 16.7 percent for the previous quarter's farrowings and 26.8 percent for expected farrowings for the next quarter. Although this study was very limited in scope, the results illustrate that the assumption of no response errors is most likely not valid for many farm items.

In view of the response inconsistencies for farm acres operated during the 1981 reinterview study, the sampling variance may be a misleading measure of precision for the weighted estimates from the JES. In most cases, the sampling variance from the weighted estimator is smaller than the other two area frame estimators, namely, the farm and tract estimators. However, the total variance from the weighted estimator may not be smaller than the total variance for the farm and/or tract estimator. Therefore, when evaluating weighted estimates, the analyst should be aware that the coefficients of variation (C.V.'s) based solely on sampling variance may seriously underestimate the C.V.'s based on the total variance for some items.

CONCLUSIONS AND RECOMMENDATIONS

The same value for "farm acres operated" was reported on the JES and reinterview study by only 31 percent of the operators in the three states tested. Also, about one-third of the reinterview responses for farm acres operated were not even within 10 percent of the JES acreage. The responses were inconsistent even when the respondent or enumerator was the same for both interviews. The component of farm acres operated that had the most inconsistent responses was land rented from others.

Despite the response discrepancies, the weighted estimates from the two interviews were not significantly different in most instances. The impact of the total response variance on the total variance is not known and may not be trivial.

Even though the inconsistency of the responses generally did not significantly affect the weighted estimates, the lack of repeatability of responses still represents a source of inaccuracy that should be minimized. Remedial measures should be considered to improve the quality of the data for farm acres operated if the use of the current weighting method is to continue. More effort should be devoted to this section of the JES questionnaire not only at the regional and state training schools but also during the interview with the respondent. Data Collection Branch should solicit input from state survey statisticians and enumerators on possible improvements to the questionnaire design and interviewing technique concerning the acres operated section. If an alternative approach is developed, a study could then be designed to test whether improvements in the data quality are obtained.

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SECTION D – ACRES OPERATED

Refer to Face Page for Type of Operation

- Individually } Go to item 1.
- Partnership or Joint }
- Managed Land - Go to item ③

1. Now I would like to ask you about the total acres you operate under this land arrangement. Include all cropland, woodland, pastureland and wasteland.

How many acres do you:

a. Own? 901

b. Rent from others? 902

c. Rent to others? 903

Then the total land you operate is (items a + b - c) 900

This total land operated consists of how many:

Cropland Acres	Farmstead, Feedlot, Pasture, & Grazed Woodland	Continuous Waste, Woods (Excluding grazed woodland) Roads, Ditches, etc.
<input style="width: 80px; border: 1px solid black; text-align: right; padding-right: 5px;" type="text"/> 903	<input style="width: 80px; border: 1px solid black; text-align: right; padding-right: 5px;" type="text"/> 906	<input style="width: 80px; border: 1px solid black; text-align: right; padding-right: 5px;" type="text"/> 907
		(Go to Section E)

③ Now I would like to ask you about the total acres you operate as a hired manager.

How many acres do you operate as a hired manager? 904

(Complete codes 903, 906 and 907 above, then go to Section E.)

APPENDIX B

Replication is a random division of a sample which provides a simple random sample of all possible samples regardless of how complex the sample design is. For complex survey designs, replication greatly simplifies the calculation of standard errors, assures equal cell sizes in the analyses of variance and yields distributions that are fairly normal. These benefits of replication make hypothesis testing from a complex survey design much more straightforward and simpler when the sample is randomly allocated to replicates.

Because of the complex survey design in this study, each JES segment in the reinterview sample was randomly assigned to one of ten replicates within each JES land use stratum in a state. The mean value for a particular replicate in a state is given by:

$$\bar{x}_{sr} = \frac{\sum_{i=1}^{n_{sr}} e_{sri} x_{sri}}{\sum_{i=1}^{n_{sr}} e_{sri}}$$

where e_{sri} is the expansion factor for the i^{th} farm operation in replicate r in state s ,

x_{sri} is the value of the item of interest for the i^{th} farm operation in replicate r in state s , and

n_{sr} is the number of farm operations in the r^{th} replicate and state s .

The sample estimates of the mean and the standard error for a given state are, respectively:

$$\bar{x}_s = \sum_{r=1}^{10} \bar{x}_{sr} / 10$$

and

$$SE(\bar{x}_s) = \left[\frac{\sum_{r=1}^{10} (\bar{x}_{sr} - \bar{x}_s)^2}{10(9)} \right]^{1/2}$$

Finally, the sample estimates of the mean and standard error aggregated over the three states are:

$$\bar{x} = \frac{10}{\sum_{r=1} \bar{x}_r} / 10$$

and

$$SE(\bar{x}) = \left[\frac{\sum_{r=1}^{10} (\bar{x}_r - \bar{x})^2}{10(9)} \right]^{1/2}$$

$$\text{where } \bar{x}_r = \frac{\sum_{s=1}^3 \sum_{i=1}^{n_{sr}} e_{sri} x_{sri}}{\sum_{s=1}^3 \sum_{i=1}^{n_{sr}} e_{sri}}$$