Forecasting Final Corn Grain Yield per Plant With a Constrained Logistic Growth Model

Greg A. Larsen

Statistical Research Division

Economics, Statistics, and Cooperatives Service

U.S. Department of Agriculture

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ABSTRACT

The report describes the use of a constrained logistic growth model for forecasting mean dry grain weight per plant for corn at the state level. The constraining procedure uses historic information to restrict the early-season parameter estimates to correspond more closely to values that would have a greater probability of occurring at maturity. The use of prior information leads to large improvements in forecasting performance. State-level forecasts on August 1 were within 9% of the final direct estimates. The September 1 forecasts are within 3% of the final direct estimate for the data sets examined.

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INTRODUCTION

The purpose of this paper is to describe the performance of a constrained logistic growth model in forecasting mean dry grain weight per plant at harvest for corn. The constraining procedure used information from previous years to restrict the early-season parameter estimates to conform more closely to those that would be obtained at the end of the season. State-level forecasts and estimates of mean grain weight per plant were made using 1977 Illinois and Iowa data sets. In this year, the constrained model produced August 1 forecasts with 9% of the final and September 1 forecasts within 3% of the final direct estimates for both states.

DATA COLLECTION

The data were collected from a subsample of the fields used in the USDA-ESCS corn objective yield forecasting and estimating program. The sample consisted of 80 fields in each state selected with probability proportional to acreage. Two randomly and independently located units were laid out in each field the week of June 30. Each unit contained 100 consecutive tagged and numbered plants in a single row.

Observation of silking began when the units were located and continued at weekly intervals until the week of August 12 when virtually all plants had silked. Some "mid-week" silking visits were made during the periods of heaviest silking. Each tagged plant was classified as silked on the visit when one or more silked ear shoots were observed. A silked plant was given another tag of a different color to readily distinguish it from plants which had not yet silked.

Weekly ear sampling began no earlier than July 15. Sampling was initiated when 50% or more of the tagged plants in a field had silked or on July 29, whichever came first. In most fields, ear sampling began on July 15. A random set of eight plants per unit was designated for ear sampling on each visit until harvest. All ears and silked ear shoots from the first four silked plants in the designated random set were husked, bagged, and tagged to identify the plant and sent to a laboratory for processing.

In the lab, kernel rows were counted and a random row was extracted from each ear. The kernels were dried for 72 hours at approximately 150°F to remove all but an estimated 1.8% of the moisture. The dried kernels were weighed and the mean grain weight per plant was estimated based on weight of kernels per row and rows per ear for the plant(s).

An estimated date of silk emergence was calculated for each silked plant by averaging the date of the last visit prior to silk observance with the date of the visit on which silking was first observed. The mean time since silk emergence and the mean grain weight per plant were calculated for all plants that were sampled from the same field on the same visit. The data were aggregated to the field level by ear sampling visit.

A more complete description of the data collection can be found in a report on the original study by Rockwell, 1978.

THEORY

Logistic Growth Model

The logistic growth model has been used in previous research to describe the time-growth relationship during grain filling for corn and wheat. Several reports which make use of this model are listed in the reference section. In the case of corn, the logistic growth model has been used to describe the relationship between mean grain weight per plant and time since silk emergence. The basic form of growth model is as follows:

(1) $$y_i = \frac{\alpha}{1 + \beta\rho^t} + \varepsilon_i$$ (where t is the value of time associated with the ith observation)

$\alpha > 0$, $\beta > 0$, $0 < \rho < 1$,

y_i = dependent growth variable,

t = independent time variable,

ε_i = error term.

Least squares theory was used to estimate the parameters α, β, and ρ. This requires the following assumptions about the nature of the model:

(2) $E(\varepsilon_i) = 0$ for all i,

(3) $Var(\varepsilon_i) = E(\varepsilon_i^2) = \sigma^2$ for all i,

(4) $Cov(\varepsilon_i, \varepsilon_j) = E(\varepsilon_i \varepsilon_j) = 0$ for all $i \neq j$.

The parameter which we are most interested in estimating is the asymptote, α. The asymptote is the mean grain weight per plant when the time variable is very large. For corn, it is usually not necessary to truncate the model and obtain an estimate of mean grain weight at harvest. This is because most of the grain is made several weeks prior to harvest and the model is usually close to convergence at the time of harvest.

Because the parameters α, β, and ρ are correlated, an understanding of their roles in the model is necessary to obtain reasonable forecasts of the final $\hat{\alpha}$. If α and β are held constant, increasing ρ causes the curve to expand horizontally without changing the y-intercept. This implies that the higher the value of ρ, the longer it takes for convergence and, hence, the flatter the curve. To get the characteristic shape of the growth curve in Figure 1, ρ is constrained to be some value between zero and one. For values of ρ equal to zero and one, the growth model produces horizontal lines at $y=\alpha$ and $y = \alpha/(1+\beta)$, respectively.

If α and ρ are held constant, increasing values of β produce a horizontal shift to the right. The asymptote stays the same while the y-intercept approaches zero as β goes to infinity. This implies that the rate of growth as evidenced by the slope of the tangent at the point of inflection is independent of the value of β. At $\beta = 1$, the point of inflection coincides with the y-intercept. β is constrained to be a positive value so that the curve is in the first quadrant. When $\beta = 0$, a horizontal line is produced at $y = \alpha$.

If β and ρ are held constant, increasing values of α produce a vertical shift up. However, unlike β, the slope of the inflectional tangent increases while the inflection point itself remains at the same value of t. This can be seen since as α increases, the y-intercept $(\alpha/(1+\beta))$ increases more slowly than does the asymptote which is α itself. This is true for any values of β and ρ within the previously mentioned constraints. α is constrained to be a positive value.

The field-level mean values used as input into the growth model were weighted by the estimated plant population per acre in each field. This weighting was to account for any relationship between plant density and grain weight per plant.

Adjusted Logistic Growth Model

In practice, the variance of the mean grain weight per plant is not constant over time. This can be clearly seen in Appendix Figures 3 and 6 where the variance is much smaller for small values of time since silk emergence than for large values. This situation is a violation of the assumption in (3) and may cause unreliable parameter estimates in some cases. An adjustment may be used so the assumption is not violated.

The assumption in (3) and (4) imply that the variance-covariance matrix of the dependent variable y is

$$I_n \sigma^2 = \begin{bmatrix} 1 & 0 & . & . & . & 0 \\ 0 & 1 & & & & . \\ . & & . & & & . \\ . & & & . & & . \\ 0 & . & . & . & 0 & 1 \end{bmatrix}_{nxn} \sigma^2$$

When the variance of y is not constant over time but the errors are still uncorrelated, the variance-covariance matrix becomes

$$V_n \sigma^2 = \begin{bmatrix} k_1 & 0 & . & . & 0 \\ 0 & k_2 & & & . \\ . & & . & & . \\ . & & & . & . \\ . & & & & .. \\ 0 & . & . & 0 & k_n \end{bmatrix}_{nxn} \sigma^2$$

The assumption in (3) can then be stated as:

(5) $E(\varepsilon_i^{\,2}) = k_i \sigma^2 = \sigma_t^{\,2}$ (where t is the value of time associated with the ith observation).

The basic model in (1) is divided by $\sqrt{k_i}$, the adjusted model becomes

(6) $$\frac{y_i}{\sqrt{k_i}} = \frac{\alpha}{\sqrt{k_i}\,(1 + \beta\rho^t)} + \frac{\varepsilon_i}{\sqrt{k_i}}$$

and it can be seen that the error term has constant variance over time.

$$E((\varepsilon_i/\sqrt{k_i})^2) = E\,(\varepsilon_i^{\,2})/k_i = \sigma^2 .$$

The problem of adjusting for unequal variance then becomes one of using appropriate values for k_i. If σ_t^2 is as defined in (5), a value of k_i equal to σ_t^2/σ^2 would produce a constant variance of σ^2 while a value of k_i equal to σ_t^2 would produce a constant variance of one. In either case, the variance would be constant over time and the absolute value of the residuals would appear to have a random relationship with time. The use of the first alternative is discussed in some detail in an earlier report (Larsen, 1978). The second alternative has been labeled the standard error adjustment and used in several previous reports (House, 1978, 1979; Rockwell, 1978). While the methods used to estimate the alternative values of k_i have differed somewhat, the parameter estimates would be identical with either if σ^2 and σ_t^2 were known. The standard error adjustment was used here for comparability with the earlier corn research.

A step function can be used to estimate the unknown continuous relationship between the variance of the grain weight per plant, σ_t^2, and time. The variance was estimated in each two-day interval of time using a weighted sum of squared residuals from an unadjusted fit of the model in (1). The weights used were plants per acre. For this adjustment to produce residuals that are uncorrelated with time, the Mean Square Error (MSE) in the adjusted model needs to be near one. If the adjusted regression accounts for a different amount of the total variability than the unadjusted regression, the MSE will differ from one. This is particularly a problem with the constrained logistic model because it generally does not fit the data as well and the MSE is correspondingly higher. Therefore, applying the standard error adjustment may cause a large change in the variability associated with the regression and the adjustment may not produce residuals which are uncorrelated over time.

In addition to the standard error adjustment, a so-called double standard error adjustment was used. The σ_t^2 values were re-estimated using the residuals from the first adjusted fit and the adjusted model in (6) was run a second time.

Constrained Logistic Growth Model

A constrained growth model was used in previous research to improve early-season forecasts of winter wheat yields (Larsen, 1979). The procedure takes advantage of information from prior years to restrict parameter estimates so that early-season forecast accuracy may be improved by giving a higher probability to values of the parameter near the historic mean value. The constrained model makes use of an inflection point relationship. This is the point at which the slope of the tangent to the curve is at a maximum. For values of time before the inflection point, the growth rate is accelerating while the growth rate slows down after the inflection point. The point of inflection is found by setting the second derivative with respect to t equal to zero and solving for t. When this is done, t is found to be $-(\ln\beta)/(\ln\rho)$. So, the inflection point is determined by the values of β and ρ.

The reason for using the inflection point to constrain the parameter estimates is that it was observed in several data sets over years that the departure of the forecasted inflection point from the final inflection point corresponded closely with the direction of the forecasted $\hat{\alpha}$ from the final α. It was also observed that the final inflection point is quite consistent over years and states. Therefore, a historic inflection point is used to force the inflection point during early-season forecasts to be close to the final end of season inflection point. This generally produces $\hat{\alpha}$ forecasts which are closer to the final $\hat{\alpha}$. The historic inflection point is a random variable with an associated variance; however, this paper treats it as a constant. One way to reflect this variability in the $\hat{\alpha}$ forecasts would be to construct a confidence interval and calculate a forecast for the upper and lower bounds.

From the second derivative of the logistic model in (1), the inflection point is

$$(7) \quad IP = \frac{-(\ln\beta)}{(\ln\rho)}$$

Solving for β, gives

$$(8) \quad \beta = \rho^{-IP}$$

Substituting (8) in (1) gives the constrained logistic growth model.

$$(9) \quad y_i = \frac{\alpha}{1 + \rho^{(t-IP)}} + \varepsilon_i$$

Now, only two parameters are estimated by least squares since β is completely determined by the value of ρ and the historic inflection point. A further constraint used in (9) is that ρ is restricted by a range of values obtained in prior years.

RESULTS

The logistic growth model in (1) and the constrained model in (9) were both applied to the 1977 Illinois and Iowa corn data. Both models were fit with and without an adjustment for unequal variance over the range of time. The prior information to use the constrained model came from 1975 and 1976 Iowa data sets and is shown in Table 1. The mean inflection point from the unadjusted models is 31.6 and the standard error adjusted models is 31.5.

Table 1: Historic Inflection Points (IP) and Values of $\hat{\rho}$

Data Set	Unadjusted		S.E. Adjusted	
	IP	$\hat{\rho}$	IP	$\hat{\rho}$
1975 Iowa	31.3	.893	32.1	.895
1976 Iowa	31.9	.892	30.8	.873

Table 2 shows how the various growth model forecasts compared to the direct estimate of mean dry grain weight per plant at harvest. The direct estimate is a weighted mean of the dry grain weights obtained on the final preharvest visit. This estimate is based on the weight of kernels in each sampled row times the number of rows per ear for each ear on the plant. The unconstrained sections of the table correspond to the basic model in (1) with and without adjustment for unequal variance. The constrained sections correspond to the model in (9). Adjustments are indicated by U (unadjusted), SE (standard error adjustment) and DSE (double standard error adjustment). The stalk population weights were applied consistently throughout all the growth models. The standard error adjustments were based on weighted residuals. A check mark (✓) follows the forecast which was closest to the final direct estimate within each state.

For the August 1 forecast, the unadjusted constrained forecast was closest to the direct estimate in both states. The improvement from the constraining procedure was large in Iowa. For September 1, the double standard error adjusted constrained forecast was closest to the final estimate in Illinois. In Iowa, the unadjusted unconstrained model produced the best estimate. In general, the constraining procedure produces the largest improvement in forecasting accuracy early in the season because the number of data points are relatively few. As more data becomes available, the constrained model with the historic inflection point should be replaced by the unconstrained model. The constrained model is preferred through the September 1 forecast.

Another result from Table 2 is that the unadjusted models generally produce better forecasts of the direct estimate than the adjusted models. It was stated earlier that the unequal variance of mean dry grain weight over time <u>could</u> cause unreliable parameter estimates. In this case, it apparently did not. This suggests that great care should be taken in assuming that the adjusted model is preferred.

Table 2: Growth Model Forecasts of Dry Grain Weight Per Plant

Lab Week	Model Form	ILLINOIS Unconstrained $\hat{\alpha}$ grams	ILLINOIS Unconstrained % of Dir.Est.	ILLINOIS Constrained $\hat{\alpha}$ grams	ILLINOIS Constrained % of Dir.Est.	IOWA Unconstrained $\hat{\alpha}$ grams	IOWA Unconstrained % of Dir.Est.	IOWA Constrained $\hat{\alpha}$ grams	IOWA Constrained % of Dir.Est.
3 (Aug. 1)	U	133.3	85.4	168.5(√)	107.9	97.5	69.7	151.3(√)	108.2
	SE	111.2	71.2	185.9	119.1	96.1	68.7	169.2	121.0
	DSE	108.6	69.6	180.0	115.3	96.2	68.8	161.1	115.2
7 (Sept. 1)	U	150.0	96.1	158.5	101.5	139.3(√)	99.6	143.3	102.5
	SE	141.2	90.5	157.3	100.8	128.9	92.2	144.3	103.2
	DSE	136.4	87.4	156.9(√)	100.5	124.9	89.3	144.8	103.6
Final Direct Estimate (grams):		156.1				139.8			

Tables 3 and 4 summarize additional information on how the constrained model compares to the basic growth model. R_p is the Pearson product-moment correlation between the absolute value of residuals and time since silk emergence. The significance probability of this correlation is an indicator of the degree of unequal variance over the range of time. The nonparametric Spearman correlation was also calculated but is not presented because, in most cases, it did not differ enough from the Pearson correlation to change the statistical significance level. In these tables, the $\hat{\alpha}$ forecasts are compared with the final $\hat{\alpha}$'s. While this is not as meaningful in a forecasting sense as a comparison to the final direct estimate, it does give insight into model behavior. With all the data available, the basic growth model should be used since it does not force the equation through what may be an incorrect inflection point. With this in mind, the "% of final $\hat{\alpha}$" column values are calculated relative to the unconstrained unadjusted and double standard error adjusted $\hat{\alpha}$'s with all the data.

From Tables 3 and 4, it can be seen that the constrained model improves the forecasts of the final $\hat{\alpha}$ in all cases. The unadjusted forecasts from the constrained model are closer to the corresponding final $\hat{\alpha}$ than are the adjusted forecasts. The forecasts in Iowa are generally closer to the final $\hat{\alpha}$ than those in Illinois.

All these observations are at least partly related to the historic inflection point that was supplied to the constrained model. The historic unadjusted inflection point of 31.6 compared favorably with the final of 31.2 in Illinois and 32.2 in Iowa. However, the historic adjusted inflection points of 29.7 in Illinois and 30.2 in Iowa. The adjusted forecasts would have been closer to the final adjusted $\hat{\alpha}$ if the historic inflection point had been closer to the 1977 point of inflection.

It is possible that the historic information should have been better. The adjusted inflection point came from a single standard error adjustment. In the 1976 Iowa data set, the residuals were significantly correlated with time after the adjustment. Documentation as to the success of the adjustment in the 1975 data could not be located. In Tables 3 and 4, it can be seen that the single standard error adjustment still left correlations which were significant at the .01 level with 7 and 12 weeks of data. The final inflection points decreased when the double standard error adjustment was used and are quite close for the two states. If the standard error adjustment had been completely successful in the 1975 and 1976 data sets, the prior inflection point may have been closer to the 1977 final. It can be seen in Tables 5 and 6 in the Appendix that the historic information on ρ was very close to the final ρ values.

Had the historic inflection points been exactly equal to the final inflection points, the forecasts would have still been higher than the final $\hat{\alpha}$'s. The reason for this is that some of the plants silked after

Table 3: Illinois Forecasts of the Final $\hat{\alpha}$

Lab Week	Model Form	Unconstrained				Constrained			
		$\hat{\alpha}$ grams	% of Final $\hat{\alpha}$	R_P	IP	$\hat{\alpha}$ grams	% of Final $\hat{\alpha}$	R_P	IP
2	U	1000+	1000+	.63**	86.5	170.4	108.6	.42**	31.6
	SE	130.6	87.6	.12	24.2	192.3	129.0	.33**	31.5
	DSE	114.7	76.9	.03	23.1	174.5	117.0	.20*	31.5
3 (Aug 1)	U	133.3	85.0	.57**	26.5	168.5	107.4	.36**	31.6
	SE	111.2	74.6	.12	24.3	185.9	124.7	.27**	31.5
	DSE	108.6	72.8	.04	24.0	180.0	120.7	.00	31.5
5	U	136.0	86.7	.54**	27.9	164.1	104.6	.45**	31.6
	SE	123.6	82.9	.16**	26.4	165.7	111.1	.24**	31.5
	DSE	118.6	79.5	.05	25.8	167.6	112.4	.11*	31.5
7 (Sept 1)	U	150.0	95.6	.49**	30.1	158.5	101.0	.45**	31.6
	SE	141.2	94.7	.20**	28.8	157.3	105.5	.13**	31.5
	DSE	136.4	91.5	.09	28.0	156.9	105.2	.05	31.5
12 (All Data)	U	156.9	100.0	.48**	31.2	158.2	100.8	.48**	31.6
	SE	151.4	101.5	.14**	30.1	157.3	105.5	.11*	31.5
	DSE	149.1	100.0	.07	29.7	156.9	105.2	.05	31.5

** significant at .01 level.
 * significant at .05 level.

Table 4: Iowa Forecasts of the Final $\hat{\alpha}$

Lab Week	Model Form	Unconstrained $\hat{\alpha}$ grams	Unconstrained % of Final $\hat{\alpha}$	Unconstrained R_P	Unconstrained IP	Constrained $\hat{\alpha}$ grams	Constrained % of Final $\hat{\alpha}$	Constrained R_P	Constrained IP
2	U	614.0	414.3	.57**	38.7	136.4	92.0	.44**	31.6
	SE	94.7	67.1	.06	23.4	160.1	113.4	.32**	31.5
	DSE	83.1	58.9	.04	22.3	150.6	106.7	.08	31.5
3 (Aug. 1)	U	97.5	65.8	.60**	24.1	151.3	102.1	.52**	31.6
	SE	96.1	68.1	.02	23.9	169.2	119.8	.31**	31.5
	DSE	96.2	68.1	.02	23.9	161.1	114.1	.06	31.5
5	U	137.6	92.8	.57**	29.9	149.6	100.9	.54**	31.6
	SE	115.5	81.8	.15**	26.8	152.2	107.8	.20**	31.5
	DSE	109.4	77.5	.04	25.9	153.8	108.9	.02	31.5
7 (Sept. 1)	U	139.3	94.0	.55**	30.8	143.3	96.7	.54**	31.6
	SE	128.9	91.3	.16**	28.7	144.3	102.2	.12**	31.5
	DSE	124.9	88.5	.04	27.9	144.8	102.5	.01	31.5
12 (All Data)	U	148.2	100.0	.47**	32.2	146.3	98.7	.47**	31.6
	SE	144.0	102.0	.12**	30.9	146.0	103.4	.10*	31.5
	DSE	141.2	100.0	.03	30.2	146.1	103.5	.01	31.5

** significant at .01 level.

* significant at .05 level.

July 15 and were not sampled as heavily. The later silking plants had somewhat less weight per ear and were from units with fewer plants per acre. As a result, the forecasts with two and three weeks of data are high because they are based on the earlier and, in 1977, higher yielding fields.

From Tables 2-4, it can be observed that the constrained model produced consistently better forecasts of the final $\hat{\alpha}$ value, but this did not necessarily mean that the constrained model was always better in terms of forecasting the final direct estimate. If the final growth model estimate is different from the final direct estimate, the constrained model may be inferior to the unconstrained model for some forecasts. Also, the fact that the historic adjusted inflection point was not particularly close to the final point of inflection may have been the reason the unadjusted forecasts were generally preferred in Table 2.

It should be pointed out that the 1977 corn crop was much earlier than normal in Iowa and Illinois. The degree of success in using the logistic growth model would be diminished in normal or late years. In a normal year, there is typically five weeks of data available for a September 1 forecast. With five weeks of data, the best constrained forecasts were 5% and 7% above the final direct estimate in Illinois and Iowa, respectively. The unconstrained model produced corresponding forecasts which were 13% low in Illinois and 2% low in Iowa.

CONCLUSION

The constrained logistic growth model generally produced better forecasts of the direct estimate of mean dry grain weight per plant at harvest than the corresponding unconstrained model. Improvements were large at the early forecast dates.

Adjustments were made to the model to correct for the affect of unequal variance of dry grain weight over time. However, for 1977, the unadjusted models generally produced the better forecasts of mean dry grain weight per plant at harvest.

Using the unadjusted constrained model, the August 1 forecast was 7.9% higher than the final direct estimate in Illinois and 8.2% high in Iowa. The September 1 forecast was 1.5% high in Illinois and 2.5% high in Iowa.

The 1977 season was earlier than normal so the results were better than would be expected in a typical year. Generally, there is almost no data available for an August 1 forecast and five weeks of data are available for September 1. With five weeks of data, the unadjusted constrained model produced forecasts which were 5.1% high in Illinois and 7.0% high in Iowa.

Although overshadowed somewhat by the forecasting performance of the unadjusted models in this particular year, it should be noted that a double standard error adjustment was successfully used to produce residuals whose absolute values appeared to be uncorrelated with time. The single standard error adjustment which has been used in the past sometimes failed to completely adjust for the unequal variance. The single adjustment was particularly ineffective when the constrained model was used.

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APPENDIX

Tables 5 and 6 summarize the parameter and variance estimates at different forecast dates for the two states. Some of the columns which have not appeared in previous tables need explanation. The "MSE" column gives the mean square error from the nonlinear regression error sum of squares. In the unadjusted models, the MSE is an estimate of the true population variance, σ^2. In the adjusted models, the MSE is an estimate of σ^2/σ_t^2 where σ_t^2 is the true variance at t days since silking. The adjusted MSE's should be near one if they are good estimates. The "unadjusted MSE" column gives the estimate of the population variance for each model. In addition to the parameter estimates for α,β and ρ, the relative standard error (RSE) expressed in a percent is given. The RSE is calculated by dividing the standard error of the estimate by the parameter estimate itself. A RSE is not given for $\hat{\beta}$ in the constrained model because $\hat{\beta}$ is calculated directly from $\hat{\rho}$.

Figures 1-6 show plots of the aggregated data and the estimated growth curves at different dates. The figures show how the unconstrained and constrained models fit the data for each state. The unconstrained model is graphed as a solid line with dots and the constrained as a solid line. On all figures, A indicates one observation, B two observations, C three observations, etc.

Table 5: Illinois Parameter and Variance Estimates

Unconstrained Logistic Growth Model

Lab Week	Model Form	Obs.	R^2	MSE	Unadj. MSE	$\hat{\alpha}$	RSE (%)	$\hat{\beta}$	RSE (%)	$\hat{\rho}$	RSE (%)
	U	115	---	---	---	1000+	---	---	---	---	---
2	SE	115	.928	.940	44.6	130.6	25.5	121.5	18.8	.820	1.2
	DSE	115	.925	1.02	46.0	114.7	20.8	113.5	15.4	.815	1.2
	U	176	.942	69.3	69.3	133.3	11.8	81.8	14.5	.847	1.3
3	SE	176	.948	.937	70.3	111.2	7.4	92.7	7.2	.830	.7
	DSE	176	.945	1.02	70.7	108.6	7.0	94.9	6.9	.827	.7
	U	300	.962	154.8	154.8	136.0	3.9	57.5	14.6	.865	.8
5	SE	300	.952	.899	158.5	123.6	3.6	80.2	7.1	.847	.5
	DSE	300	.948	.999	162.4	118.6	3.6	87.0	6.0	.841	.4
	U	426	.962	300.2	300.2	150.0	2.4	41.2	14.2	.884	.6
7	SE	426	.961	.893	307.4	141.2	1.9	65.3	7.5	.865	.4
	DSE	426	.956	.977	315.4	136.4	1.9	78.8	6.1	.856	.3
	U	581	.960	507.4	507.4	156.9	1.3	36.3	14.8	.891	.5
12	SE	581	.962	.919	519.0	151.4	1.3	57.5	7.4	.874	.3
	DSE	581	.959	.980	531.0	149.1	1.2	71.1	6.1	.866	.3

Constrained Logistic Growth Model

Lab Week	Model Form	Obs.	R^2	MSE	Unadj. MSE	$\hat{\alpha}$	RSE (%)	$\hat{\beta}$	RSE (%)	$\hat{\rho}$	RSE (%)
	U	115	.857	86.4	86.4	170.4	8.4	37.0	---	.892	.7
2	SE	115	.912	.518	64.8	192.3	7.2	72.1	---	.873	.5
	DSE	115	.887	.941	78.9	174.5	8.8	72.1	---	.873	.5
	U	176	.922	91.9	91.9	168.5	3.4	37.0	---	.892	.4
3	SE	176	.942	.593	72.3	185.9	3.0	72.1	---	.873	.3
	DSE	176	.938	.995	75.5	180.0	2.9	72.1	---	.873	.2
	U	300	.959	163.8	163.8	164.1	1.2	37.0	---	.892	.3
5	SE	300	.953	.834	167.7	165.7	1.4	58.8	---	.879	.2
	DSE	300	.946	.960	177.9	167.6	1.6	72.1	---	.873	.2
	U	426	.962	302.7	302.7	158.5	1.1	35.7	---	.893	.4
7	SE	426	.960	.911	309.2	157.3	1.0	51.3	---	.883	.2
	DSE	426	.956	.977	316.5	156.9	1.1	60.9	---	.878	.2
	U	581	.960	507.2	507.2	158.2	.9	35.7	---	.893	.5
12	SE	581	.961	.935	512.5	157.3	.8	51.4	---	.882	.2
	DSE	581	.959	.983	518.3	156.9	.9	61.2	---	.878	.2

Table 6: Iowa Parameter and Variance Estimates

Unconstrained Logistic Growth Model

Lab Week	Model Form	Obs.	R^2	MSE	Unadj. MSE	$\hat{\alpha}$	RSE (%)	$\hat{\beta}$	RSE (%)	$\hat{\rho}$	RSE (%)
	U	130	---	---	---	614.0	410.1	373.6	385.4	.858	2.7
2	SE	130	.919	1.00	33.4	94.7	50.4	75.2	42.9	.831	1.6
	DSE	130	.919	1.02	33.9	83.1	44.0	69.1	36.9	.827	1.6
	U	196	.914	77.6	77.6	97.5	16.2	73.9	19.3	.837	1.9
3	SE	196	.923	1.02	77.6	96.1	15.1	74.2	11.1	.835	.9
	DSE	196	.923	1.02	77.6	96.2	15.1	74.4	11.1	.835	.9
	U	328	.916	284.6	284.6	137.6	8.0	50.5	18.7	.877	1.1
5	SE	328	.909	.896	291.0	115.5	5.5	68.8	7.2	.854	.6
	DSE	328	.913	.999	296.7	109.4	5.1	73.4	6.0	.847	.5
	U	460	.914	576.4	576.4	139.3	3.8	39.1	20.8	.888	.9
7	SE	460	.912	.889	586.2	128.9	3.0	63.5	7.7	.865	.5
	DSE	460	.914	.990	594.1	124.9	2.9	72.5	5.7	.858	.4
	U	600	.918	859.0	859.0	148.2	2.0	35.9	20.2	.895	.7
12	SE	600	.926	.915	871.5	144.0	1.7	58.1	7.4	.877	.4
	DSE	600	.923	.986	882.6	141.2	1.8	68.8	5.7	.869	.3

Constrained Logistic Growth Model

Lab Week	Model Form	Obs.	R^2	MSE	Unadj. MSE	$\hat{\alpha}$	RSE (%)	$\hat{\beta}$	RSE (%)	$\hat{\rho}$	RSE (%)
	U	130	.858	44.9	44.9	136.4	10.5	37.0	---	.892	.8
2	SE	130	.892	.617	39.4	160.1	12.3	72.1	---	.873	.7
	DSE	130	.901	.980	43.5	150.6	13.3	72.1	---	.873	.7
	U	196	.899	90.6	90.6	151.3	4.2	37.0	---	.892	.5
3	SE	196	.912	.635	80.6	169.2	4.5	72.1	---	.873	.4
	DSE	196	.908	.986	84.8	161.1	5.2	72.1	---	.873	.3
	U	328	.915	286.9	286.9	149.6	1.7	37.0	---	.892	.4
5	SE	328	.905	.820	287.2	152.2	2.1	61.8	---	.877	.2
	DSE	328	.906	.976	293.1	153.8	2.3	72.1	---	.873	.2
	U	460	.914	575.8	575.8	143.3	1.6	36.4	---	.892	.6
7	SE	460	.910	.896	586.5	144.3	1.6	55.7	---	.880	.2
	DSE	460	.911	.984	594.4	144.8	1.6	64.2	---	.876	.2
	U	600	.918	858.6	858.6	146.3	1.4	36.5	---	.892	.7
12	SE	600	.924	.916	866.4	146.0	1.2	56.2	---	.880	.2
	DSE	600	.923	.987	871.7	146.1	1.3	64.9	---	.876	.2

Figure 1
Illinois -- 3 Weeks of Data

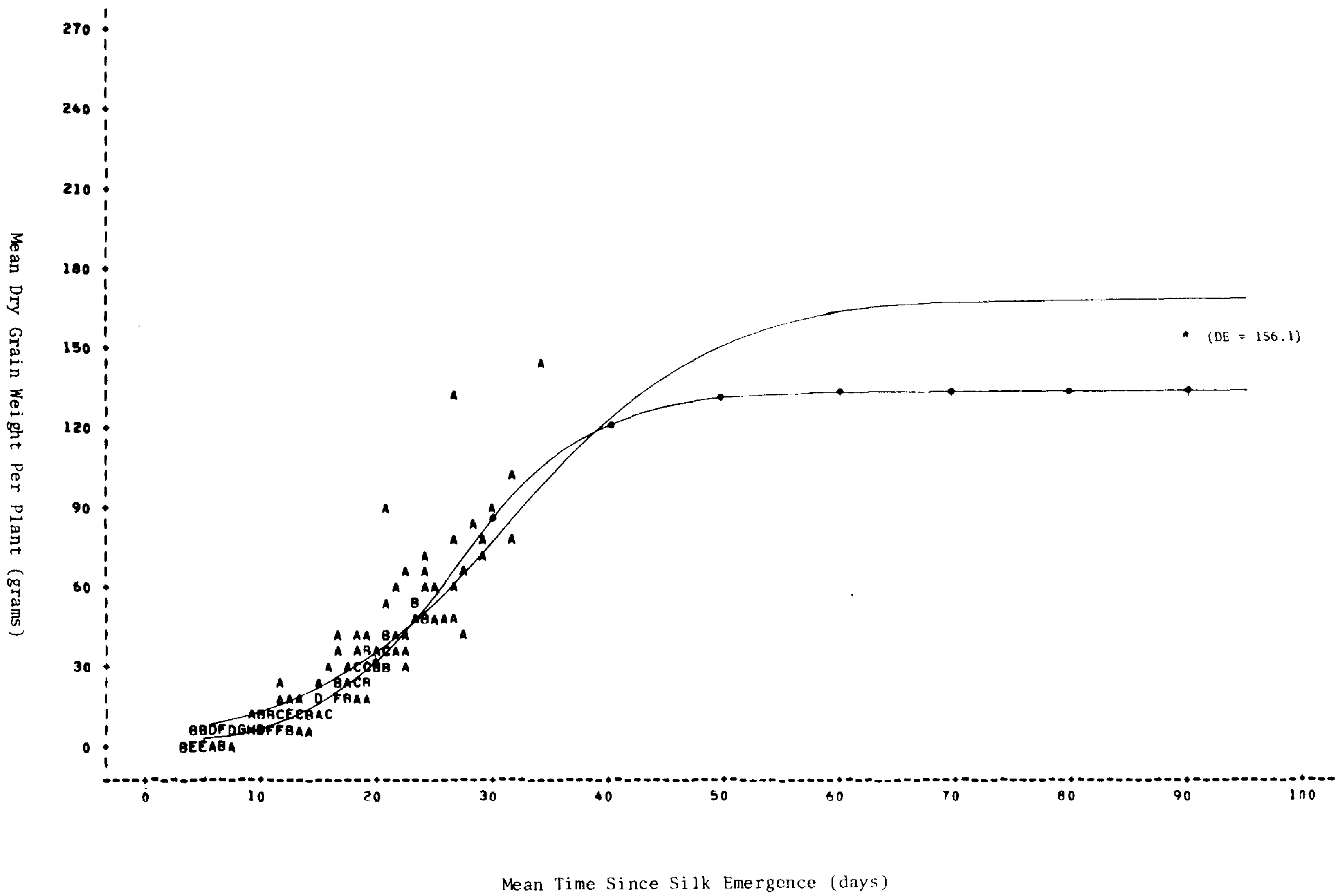

Figure 2
Illinois -- 7 Weeks of Data

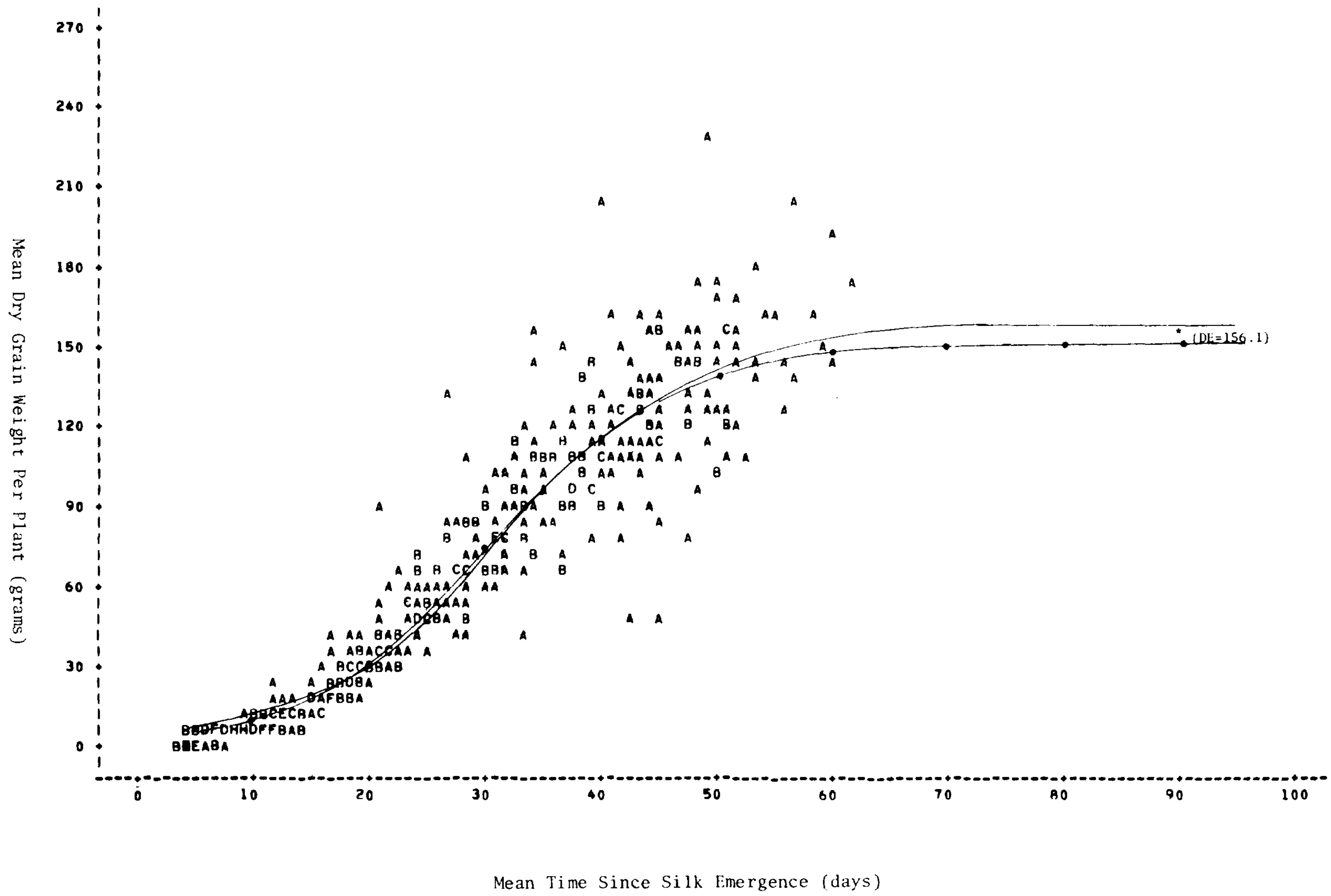

Figure 3
Illinois -- 12 Weeks of Data

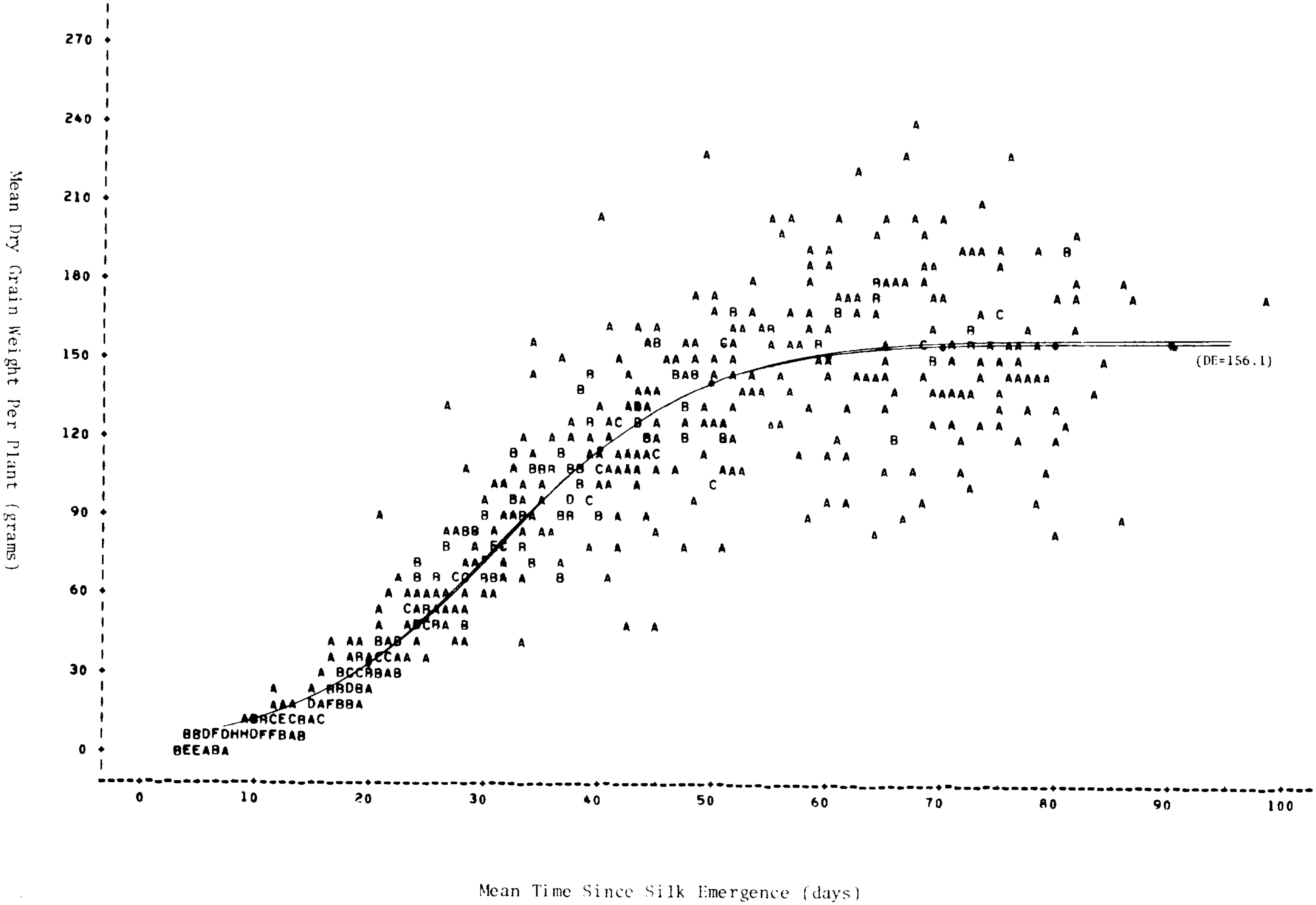

Figure 4

Iowa -- 3 Weeks of Data

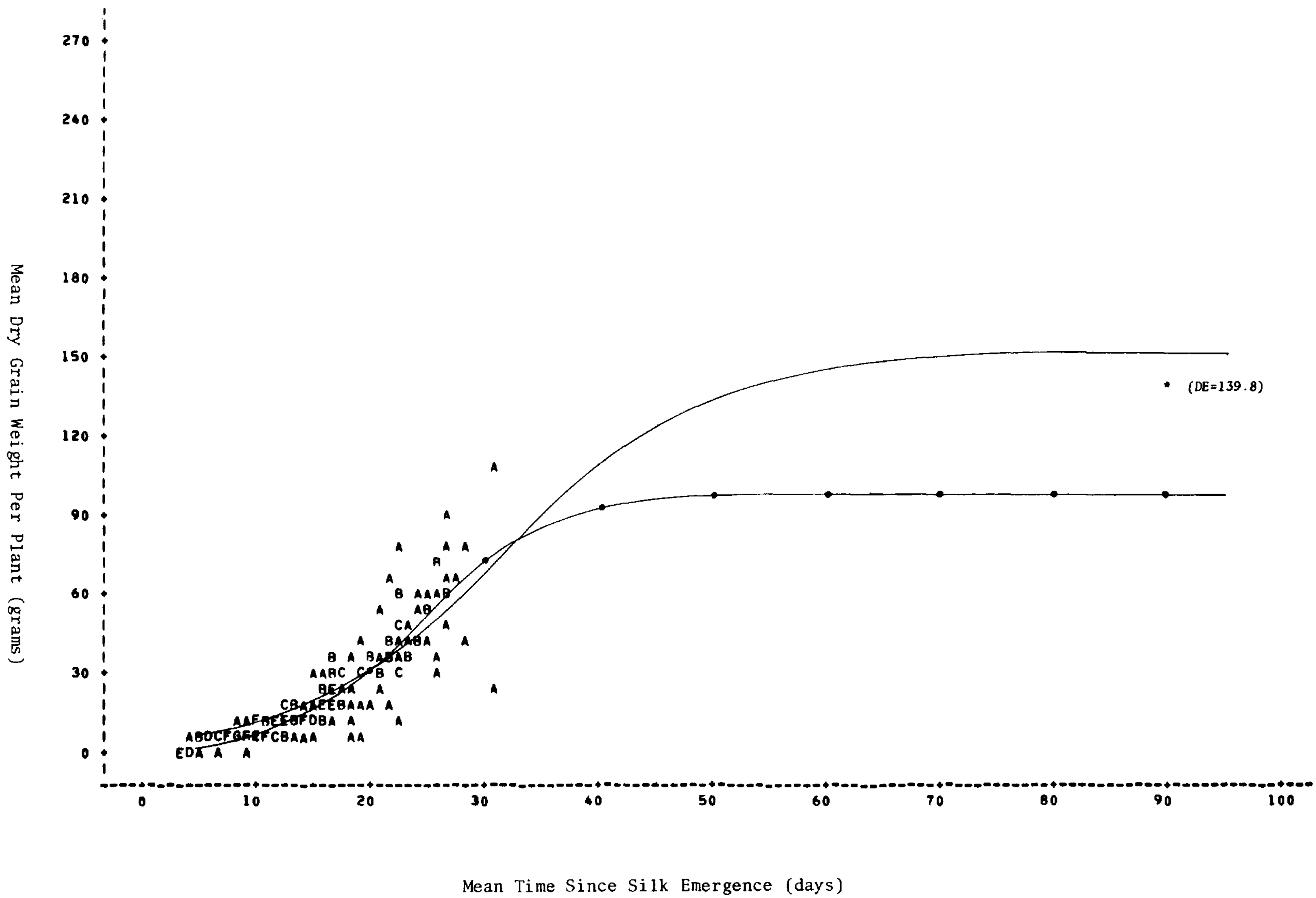

Figure 5
Iowa -- 7 Weeks of Data

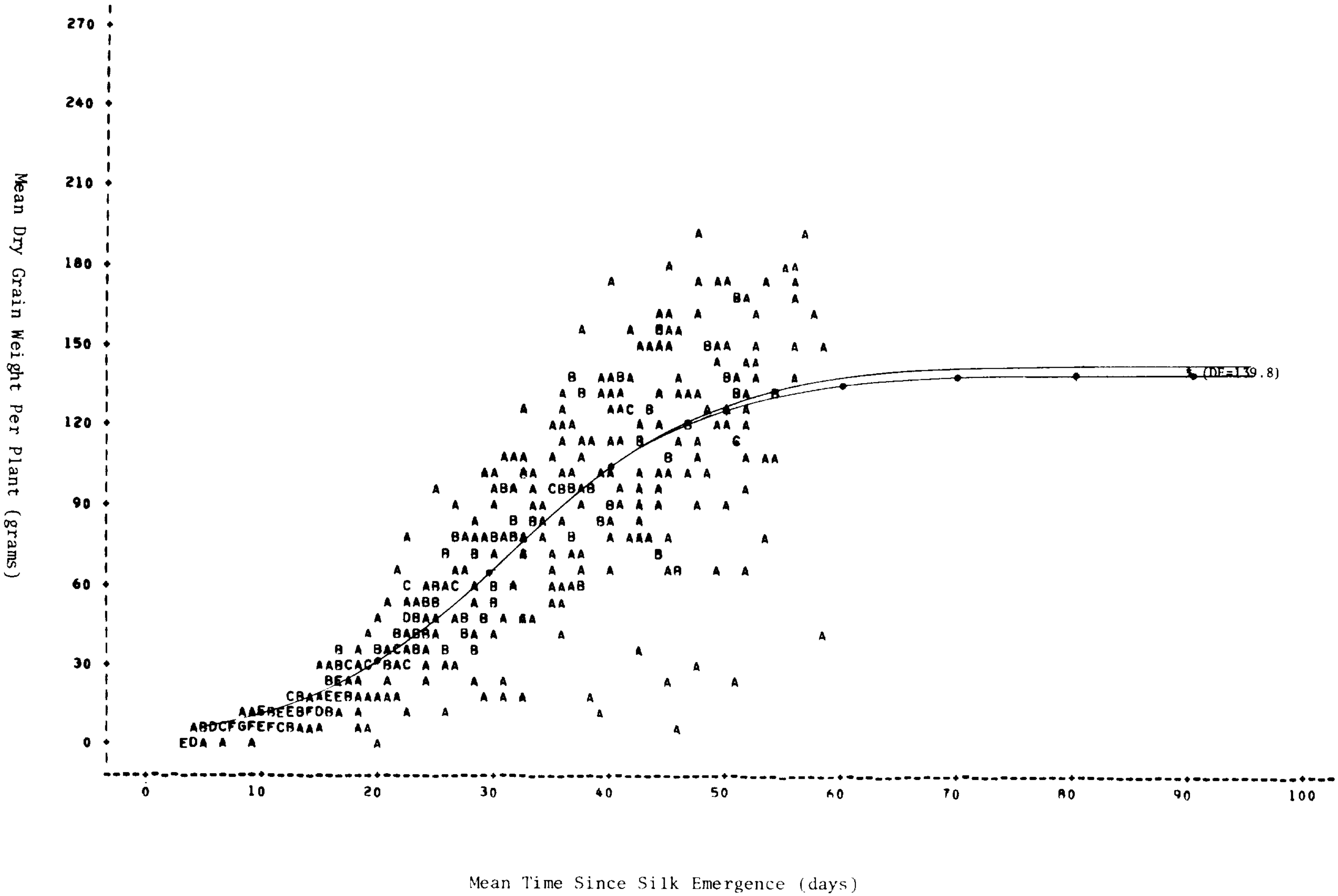

Figure 6
Iowa -- 12 Weeks of Data