PRELIMINARY REPORT ON AUTOMATED FRUIT

COUNTING FOR PHOTOGRAPHS

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Abstract

Although for a very specific case, the study demonstrates that it is possible to develop a completely automated procedure for counting oranges. This preliminary study shows that the most satisfactory technique is one on which a count of the Number of Partial Oranges seems to give the best agreement with the number of oranges counted by photo interpreters. Using the multivariate discriminant procedure in the Statistical Analysis System (SAS) and a formulated definition of an orange, 100% correct classification was obtained for mature oranges. The results for immature peaches were not as good as for mature oranges. From the analysis it is apparent that film density measurements on color Kodachrome slides based on the red filter contribute significantly for automated fruit classification schemes.

Preliminary Report on Automated Fruit Counting for Photographs

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Contents

HISTORY

The problem of determining the total number of fruit on a tree has long been of interest in crop estimation work. The first systematic approach to this problem was described by Jessen [6] in 1955. Obviously, a complete enumeration of fruit on a tree is quite an onerous and time consuming task. Thus, in order to reduce the time and cost, Jessen proposed a method of randomized branch sampling to obtain precise estimates of the total number of fruit.

Over the past decade, the Research and Development Branch, Statistical Reporting Service (SRS), United States Department of Agriculture has investigated various sampling techniques to estimate objective yields of the following fruits and nuts: oranges [1,16,23], peaches [19,25], cherries [10,14], apples [1,16,23], almonds [12], pecans [24], walnuts [13], filberts [8,21], grapefruit [1,16,23], and lemons [7]. In all the above cases field crews counted fruit on selected limbs and the objective yield estimate for a tree was then based on the reciprocal of the probability of selection.

In March 1968, Huddleston first published a preliminary research report [5] of earlier work begun in 1965 which examined the possibility of using photography to increase sampling efficiencies. Trees without leaves were photographed in an attempt to use the photograph as a sampling frame for limb selection. In addition, photography was used to directly obtain fruit counts of trees photographed on two sides using photo interpreters. However, realizing that accurate counting of fruit, even on a sample basis, by field crews was a costly operation. Von Steen and Allen [16] in 1969, and Allen in 1972 [1] further investigated the possibility of using ground photography as an auxiliary information source to forecast yields of fruit.

A third approach to the problem of estimating the number of fruit on a tree has been investigated by the Remote Sensing Group of the United States Department of Agriculture. A small part of Allen's work in 1969-70 involved using aerial photography to make fruit counts [1]. This approach cannot be considered significantly different from that of using ground photography since human photo interpreters are relied upon for the fruit counts in both cases.

In some earlier work by Von Steen, et al. [18], the relationship of aerial photography reflectance to crop yields was investigated. They found that when the optical density of the film was measured using red, green, blue, and clear filters, the clear and red filters produced the highest correlations when related to the yield indicators, but the magnitude of the correlation was too small to be useful in double sampling.

The newest method understudy for estimating yields of crops has encompassed the domains of image processing, coding, transmission, and pattern recognition as well as statistics and agriculture. The dramatic growth of technology and interest in the environmental resources area has emphasized the potentialities for analyzing large quantities of data in the form of pictures. In order to utilize the enormous collection of data, one must be able to process it in both an efficient and expeditious manner.

All previous methods of counting fruit involved considerable time and human energy. As the techniques advanced from the randomized branch sampling to the photo interpretation, the human element was continually being reduced. The current proposal is to retain all the elements of the estimator developed using the photo counts, but to eliminate the photo interpreter. It is desirable to eliminate the photo interpreter for several reasons. The first and probably most important reason is that because the fruit counting task is extremely monotonous and tedious, the accuracy of the counting varies directly with time. Furthermore, the physical radiant energy of a scene which is received by the photo interpreter will vary little from person to person, however, it is individual perceptions of the physical stimuli which interrelated physical and mental processes which will fluctuate from person to person. No automated device can be expected to make the complicated distinctions of which a human is capable, however, we can expect the decision making process to be more consistant.

This report is broken into two Parts, A and B. Part A describes an Univariate approach and Part B discusses a Multivariate approach.

PART A UNIVARIATE ANALYSIS

I. Introduction

Perhaps it is appropriate to briefly discuss what one could possibly expect from a mathematical approach to the problem of counting fruit in a picture. Bremermann [2] has estimated that even for a very coarse grid (20 x 20) the number of variations of a prototype exceeds 10^{12}, and this number rapidly escalates to 10^{16} when one considers translations, rotations, and dilations. Thus, at the present time, a table look-up solution is technologically impossible. Instead, many simple tests can be applied to reduce the number of configurations. Rather than attempting to develop a model which encompasses every detail of the object to be identified, a simpler approach is to isolate the essential features of the object and eliminate all candidates which do not possess those properties. The approach is to apply a sequence of operations, each of which retains an essential property. These operations may range from grey level filters to test which measure area, convexity, and connectivity.

In most situations, one can assume that there are a finite number of classes from which a pattern may have come, and each class can be characterized by a probability distribution of the measurements. However, in some cases, it may not be practical to specify the probability distribution of the pattern classes, and in other cases the structure of each class distribution may be known, but the parameters must be estimated from training samples.

Before we proceed to the classification task, we must decide which subset of the observed features is to be used as a set of features to represent a pattern. The ability of an optimum classifier to discriminate given classes depends upon statistical separability measures between class distributions in a given feature space. These measures then may reflect the effectiveness of a feature set. It is known that the performance of a pattern recognition system is closely related to the feature measurements taken by the classifier. Fu [3] has presented some experimental results for feature selection in multiclass pattern recognition applied to a crop classification problem. From the experimental results, one notes that it is possible for a feature subset to be almost as effective (in terms of percent recognition) as the complete feature set. Fu points out that at present there is no unique feature selection technique for all the possible pattern recognition problems. However, when class distributions can be reasonably approximated by Gaussian distributions, the parametric feature selection technique gives at least reasonable evaluation of feature effectiveness.

If one has complete freedom in the choice of features, they should be chosen such that:

1. they separate the patterns,

2. their number is as small as possible, and

3. they minimize the probability of misclassification.

II. Digitalization of Pictures

The first phase of a computerized method of counting fruit is to digitize the image. Since earlier research [16] on the use of aerial photography had not indicated an appreciable gain in accuracy or cost reduction over the ground photographs, it was decided to use the ground photographs which photo interpreters had used to make their fruit counts. This procedure involved taking a 35mm color photograph of the tree divided into quadrants, one picture per quadrant, from a distance of about 20 feet. For a more precise explanation of the procedure, see the reference "Use of Photography and Other Objective Yield Procedures for Citrus Fruit," [16].

Using a Photometric Data Systems (PDS) Model 1014-0003 Microdensitomerer System [9] each color transparency was digitized into an m x n array of picture elements X_{ij}, $i = 1,\ldots,m$ and $j = 1,\ldots, n$, each of which was quantified into one of 2^k grey levels. Such an array would require m x n k-bit words per picture to code any of the 2kmn possible pictures.

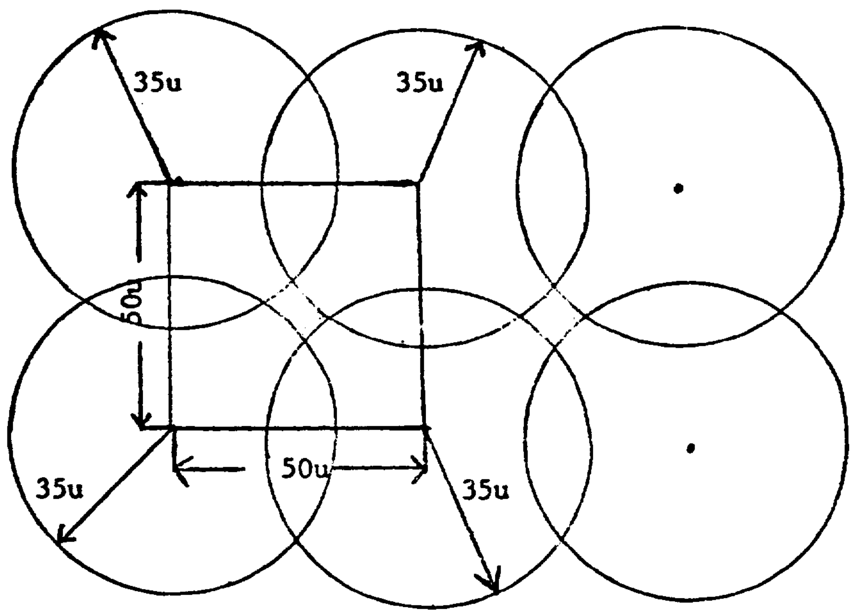
For this particular discussion, the entire 35mm transparency was scanned into a 477x715 array of $2^{12} = 4096$ grey levels. Each picture element, an analog to digital converted signal, is a 12 bit point transmission recording. With a magnification of 50 X and circular aperture of 3.45mm in diameter, an effective aperture or spot size of 70 microns was obtained. The scanning pattern was such that the distance between centers was 50 microns in both the X and Y directions.

One must remember that we are viewing a two-dimensional projection of a three-dimensional scene via a photograph, and the spectral properties of the object of interest, the fruit, are greatly affected by the position of the light source. The apparent distinct edge between the fruit and the background tree which the photo interpreter observes is really a shaded grey area when one more carefully inspects the transmission values in a neighborhood of the apparent edge. This arises because of the spherical shape of the fruit and a point light source, the sun. Hence, a circular aperture was chosen because that shape more nearly approximated the elliptical shape of the fruit than did the rectangular aperture.

The basic consideration in the choice of scanning pattern was to scan the entire transparency as efficiently as possible while still retaining the fixed effective circular aperture of 70 microns in diameter. This requirement dictated the 50 micron distance between centers in both the X and Y directions. The above pattern left only a small hole, approximately square 5×10^{-3} microns^2 in area, in the center of an array of four transmission recordings (See Figure 1). It is not at all clear that the above choice of filter size, magnification, and scanning pattern are optimum. Since those questions were not a part of this project, no further attempt will be made to discuss any optimality requirements. This area of the problem is open to further research.

In order to span the visible portion of the spectrum, the same area of the transparency was retraced three times. Each transparency was scanned once with a clear, red, green, and blue Kodak filter. Unfortunately, due to some tape handling problems, none of the transparencies in this discussion had a complete set of four readings i.e., [X'_{ij} = (clear, red, green, blue)]. Thus, rather than looking at this problem in a more general multivariate setting, we shall restrict ourselves to a univariate analysis.

Figure 1--Scanning pattern for an effective circular aperture of 70 microns in diameter. Shaded areas are exaggerated in this sketch.



III. Classification

An optimal classifier for this problem would be one that would classify each transmission measurement into one of two classes, the class of points which are fruit and those which are not fruit. One could conceivably achieve this by using only one bit per picture element. In any case, one would like to keep the number of classes as small as possible because the computing time increases rapidly with the number of classes.

In constructing a classification procedure, it is desirable to minimize the probability of misclassification. That is, a good classification procedure is one which minimizes the cost of misclassification, for a given confidence level. If one has some knowledge of his prior probabilities, the underlying distributions of the classes, and the costs of misclassification, then Bayes procedures give a method of defining "minimum cost."

Let us now give a brief description of the classes among which we wish to discriminate. First we must decide on the number of classes into which we want to classify our data. We shall approach this question from an intuitive point of view. When the human observer looks at Figure 2 there appear to be three distinct classes which have obvious spectral differences. From a gross perspective we shall identify each class by describing the major components of the class.

Class 1 is composed of the sky and dirt roadway. These areas are usually the brightest portions of the picture.

Class 2 will be our main class of interest, the fruit.

Class 3 the most popular class, is green leafy background and darker portions of the scene.

Figure 2--An orange tree showing the three classes of spectral differences sky and roadways, fruit, and green leafy background.

Because the sample size of digitized transparencies was limited and little was known about the distributions of the pattern classes, it was decided to use a simple minded nonparametric classification procedure.

During the initial scanning of the picture, training information was obtained. By scanning certain portions of the transparency which were known to be members of the i^{th} class, one was able to determine, on a sample basis, the range and distribution of the three classes. Thus, on the basis of grey level and training sample information above, one would like to construct a classification procedure of the form:

$$X = \begin{array}{lll} 0 & & X \varepsilon \ \text{Class } 1 \\ 1 & \text{if} & X \varepsilon \ \text{Class } 2 \\ 2 & & X \varepsilon \ \text{Class } 3, \end{array} \qquad (4.0)$$

and

$$\begin{array}{lll} X \varepsilon \ \text{Class } 1 & \text{if } C_0 < X = C_1 \\ X \varepsilon \ \text{Class } 2 & \text{if } C_1 < X = C_2 \\ X \varepsilon \ \text{Class } 3 & \text{if } C_2 < X = C_3, \end{array} \qquad (4.1)$$

where the C's are positive real valued integers. The only difficulty with the above procedure is that of estimating the constants C_j, $j = 0$, 1, 2, 3. Rather than having to estimate four constants as it might appear, C_0 and C_3 are lower and upper bounds respectively, which are completely determined. The lower bound C_0 is zero because it is physically impossible to obtain a negative transmission measurement. The upper bound ($C_3 = 4095$) is fixed by the number of bits which are reserved for each transmission recording. Hence, one need only estimate C_1 and C_2 from the training samples. In our problem, C_1 and C_2 were equal to 52 and 140 respectively.

IV. Analysis

Once we have classified each transmission measurement according to the grey level classification procedure, equation (4.0), we would like to obtain an estimate of the probability of misclassification. The classification procedure adopted has no theretical basis upon which we can make such an estimate, so we will give the empirically determined misclassification results for the orange tree shown in Figure 2. The results presented in Table 1 will be only part of the total solution of the problem of automatically counting the number of fruit from a transparency because our object of interest, the 'orange', is a set of points.

TABLE 1

	Point-by-Point classification summary by classes				
Class	Per Cent Correct	Number of samples classified into class 1	2	3	Number of Samples
1	33.9	2431	2907	1842	7180
2	26.3	0	1179	3309	4488
3	99.8	0	652	328735	329287
Total	97.4	2431	4738	333886	341055

The photo interpreter has a subjective mental image of an orange, a leaf, a branch, sky, dirt, and how each of these objects differ from one another. We must make the photo interpreter's perceptions of these objects more precise, by defining them in mathematically quantifiable terms, so that identification will no longer vary from interpreter to interpreter.

In order to describe the class or set of points which constitute an 'orange', we shall look at the essential properties of the element of the set.

That is, we must decide what we will accept as confirming evidence for the statement, "A is an 'orange'," where A is a particular set of points. The data which confirm a given hypothesis shall be restricted to mathematically quantifiable terms. Our idea of confirmation should also satisfy the equivalence condition. In Hempel's [4] terminology, an observation report (data) which confirms a particular sentence (hypothesis) also confirms every sentence logically equivalent to that sentence.

We now propose the hypothesis H,

> H: An 'orange' is a connected set of points (coded transmission values) of constant grey level with an elliptical boundary and major and minor axes of lengths F and G, respectively.

Here "connected set," "elliptical boundary," "major axis," "minor axis," and "constant grey level" are terms of our quantifiable observational vocabulary.

One should note that the observation reports upon which we are relying are at two levels of introspection. The first level deals with the elemental units, that is, the individual transmission values, and the second level deals with properties and relations of certain sets of the elemental units.

In the previous section on classification, we have already dealt with the observation reports which look at the data point-by-point. By using training samples on a classification procedure of the form of equation (4.0), we have made a drastic reduction in the amount of data which we will consider at our second and higher level of introspection. (See Table 2)

TABLE 2

Point-by-point Classification Results

Total Number of Points in Class		
1	2 (Fruit)	3
2431	4738	333886

We need only further consider those points which have the property namely that of being a certain "constant grey level."

Property P: A point is said to have property P if it is the "constant grey level" corresponding to the transmission values in Class 2.

This observation report directly confirms hypothesis H.

Let us now proceed to those observation reports at the second idea of confirmation that satisfies the equivalence relation, we know that an observation report B disconfirms a hypothesis K if it confirms the denial of K.

Another property Q which supports the hypothesis H is the set property of "connected set of points." Rather than look at the observation report of "connected set of points" directly, we shall look at its negation, which will provide disconfirming evidence for H. Thus, we would like to eliminate from our domain of interest those points which have property Q and do not have property P (symbolized $Q \wedge \sim P$).

Property Q: A set of points possesses Property Q if it contains horizontal runs less than length three or vertical runs less than length two.

This will eliminate all singles in the vertical direction and all singles and doubles in the horizontal direction[1/] which have property P. Those sets of points not eliminated i.e., those possessing Properties $P \wedge \sim Q$ are "connected sets of points."

A more sophisticated approach to the property of "connected set of points" is that given by Wilkins and Wintz [22]. They describe a contour tracing algorithm (CTA) which locates and traces all contours of any 2-dimensional data array. The algorithm first finds an initial or starting point and then traces the outer boundary of the largest connected set of elements having the same grey level as the initial point, always terminating at the initial point.

One could also think of defining some statistic which would give a measure of, say, the eccentricity of the contour. These techniques should give better results than our simple idea of using runs. However, due to the time involved in programming the CTA, we have postponed using those techniques until a later date.

One of the difficulties with the automated approach to the problem of counting fruit is that of keeping the notion of an 'orange' flexible enough to pick out a rather wide class of objects which satisfy the properties of an orange, but rigid enough to give consistent results. This difficulty is further complicated by the fact that many of the fruit one sees in a photograph are either partically camouflaged or else appears in clusters.

[1/] In any ordered sequence of elements of two kinds, each maximal subsequence of elements of like kind is called a run. For example, the sequence $\alpha\alpha\alpha\beta\alpha\alpha\beta\beta\beta\alpha$ opens with an alpha run of length 3; it is followed by runs of length 1, 2, 3, 1, respectively.

By eliminating runs of length one and two, we have decided not to attempt to count the very small parts of an orange which might be visible between the leaves and branches. The separate reductions for elimination of horizontal and vertical runs are given in Table 3.

TABLE 3

Data Reduction by Eliminating Short Runs in Class 2

Total Number of Points	Number of Points in Horizontal Runs of Length 1 and 2 Eliminated	Number of Points in Vertical Runs of Length 1 Eliminated	Number of Points Not Eliminated
4738	1342	1297	2099

The reduction given in Table 3 have been obtained by first eliminating horizontal runs and then vertical runs. This operation is definitely not commutative. Furthermore, one may wish to apply the run elimination procedure repeatedly until the data reduction terminates.

Since we are only interested in the set of points which are elements of Class 2, the fruit, and the set of points which are not fruit, we actually have a two class classification problem. One should note that after the short runs are removed from Class 2, they are then classified into the compliment of that set. The classification summary for the reduced two class problem after run removal is presented below in Table 4.

TABLE 4

Classification Summary After Run Removal

Class	Per Cent Correct	Number of Points Classified As		Number of Samples
		Fruit	Non Fruit	
Fruit	25.0	1122	3366	4488
Non Fruit	99.7	977	335590	336567
Total	98.7	2099	338956	341055

One can readily construct examples to prove the noncommutativity of the operation of run elimination on opposite directions for two dimensional arrays. Suppose we let h symbolize elimination of runs of length one and two in the horizontal direction, and v symbolize elimination of runs of length one in the vertical direction.

We wish to prove the following:

Theorem I: $hv \neq vh$

Proof: Let us assume the above relation does hold. Consider the following array in which we let a "2" represent a point belonging to Class 2, an "x" represent a point not in Class 2, and a "-" represent a point removed by run elimination.

$$\begin{array}{ccccccc} 2&2&2&x&2&2&2\\ 2&2&2&2&2&x&x\\ x&2&2&x&x&x&x \end{array} \xrightarrow{h} \begin{array}{ccccccc} 2&2&2&x&2&2&2\\ 2&2&2&2&2&x&x\\ x&-&-&x&x&x&x \end{array} \xrightarrow{v} \begin{array}{ccccccc} 2&2&2&x&2&-&-\\ 2&2&2&-&2&x&x\\ x&-&-&x&x&x&x \end{array}$$

$$\begin{array}{ccccccc} 2&2&2&x&2&2&2\\ 2&2&2&2&2&x&x\\ x&2&2&x&x&x&x \end{array} \xrightarrow{v} \begin{array}{ccccccc} 2&2&2&x&2&-&-\\ 2&2&2&-&2&x&x\\ x&2&2&x&x&x&x \end{array} \xrightarrow{h} \begin{array}{ccccccc} 2&2&2&x&-&-&-\\ 2&2&2&-&-&x&x\\ x&-&-&x&x&x&x \end{array}$$

It is obvious that the resulting two dimensional arrays are not equivalent. Q.E.D.

One may note in the above counterexample that by again eliminating runs in the sequence hv, we can obtain still further reduction to

2 2 2 x - - -

2 2 2 - - x x which is equivalent to vh.

x - - x x x x

A more general theorem is stated in Appendix A without proof.

The property of the set having an "elliptical boundary" is partially included in our formulation of Property $\wedge \sim Q$ and in our counting method which is described later. Thus we will not isolate the property of an "elliptical boundary," but rather will incorporate it in the observation reports of the run property and size property.

To this point, we have narrowed our domain of interest to only those sets of points which have properties P and not Q (symbolized $P \wedge \sim Q$). We now wish to devise an algorithm which will count the number of objects, 'oranges', which have the size property S, "major axis of length eight," and "minor axis of length six."

Property S: A nonempty set of points is said to have Property S if it is a "connected set of points" with area π FG/4, where F and G are the lengths of the major and minor axes.

V. Counting the Fruit

Although the final phase of accepting or rejecting the hypothesis H could be construed as pragmatic, we wish to establish general "rules of acceptance." [4] These rules would formulate fixed criteria which determine the acceptance or rejection of the hypothesis by evaluating the amount of confirming or disconfirming evidence in the accepted observation reports.

Thus, the concept of confirmation or varification of our hypothesis is relative and will depend upon some particular set of observation reports. We do not maintain that our observation reports are irrevocable, and hence do not claim to establish a unique procedure verifying the hypothesis "A is an orange."

It was noted earlier that in many instances only a portion of the fruit was visible. This difficulty did not arise from any fault of the classifier, but rather arises from the natural camouflage of the leaves, branches, and other fruit on the tree. Because of the nature of our observation reports and the physical properties of the scene, certain subsets will be more highly confirmed than others. We must, therefore, decide what to do with those sets which satisfy $P \wedge \sim Q$ and are indeed portions of oranges, but do not fully meet our size requirement of Property S.

Since it is extremely difficult to develop an efficient algorithm which is capable of locating sets which have the size property S, we have developed two approaches to the counting portion of the problem.

The first approach and probably the most simple is to sum over the entire data array all points possessing properties $P \wedge \sim Q$ and then to divide by the average number of points per 'orange,' i.e., greatest integer in π FG/4. We shall designate the estimate given by the above method to be the Number of Average Oranges (NAO). For the k^{th} tree we have,

$$NAO^{(k)} = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij}^{(k)} / [\pi FG/4], \tag{6.0}$$

where x_{ij} represents those points with properties $P \wedge \sim Q$ and [ ] is the greatest integer function.

If there was no camouflage and we had a perfect classifier, the

Average Orange (AO) approach would give us an unbasied estimate. However, the AO approach makes little use of the fact that we have several classes of oranges we are attempting to count.

The three basic classes of oranges we shall consider are:

(i) a whole single orange,

(ii) a cluster of oranges, and

(iii) a partial orange.

Case (i) presents no problems, but in cases (ii) and (iii) where only partial fruit are visible the NAO may grossly underestimate the true number of oranges. For instance, if seven partial fruit have an area equivalent to one Average Orange, we would have an undercount of six.

The motivation behind the second counting approach was to develop a simple algorithm which would search the entire data array and count partial oranges. The algorithm partitions the data matrix into "boxes" which are arrays of dimension ten rows by ten columns and checks each "box" according to the Partial Orange criterion given in equation (6.1) We obtain the Number of Partial Oranges $(NPO)_1$ in the 1^{th} "box" as,

$$NPO_1 = \sum_{i=1}^{10} \sum_{j=1}^{10} x_{ij1} \,/\, [\pi FG/12], \tag{6.1}$$

and the Number of Partial Oranges $(NPO)^k$ on the k^{th} tree as the sum of the Number of Partial Oranges over all 1 "boxes,"

$$NPO^{(k)} = \sum_{1} NPO_1^{(k)} \tag{6.2}$$

One readily observed from equation (6.1) that a Partial Orange is about one-third an Average Orange. Although the NPO approach has its shortcomings, it does represent an attempt to utilize the clustering property of sets of points substituting an 'orange.'

The estimates for the Number of Average Oranges, equation (6.0), Number of Partial Oranges, equation (6.2), and the photo interpreter counts for the orange tree shown in Figure 2 are presented in Table 5.

TABLE 5

Number of oranges in lower right quadrant of Figure 2 as determined by three different counting methods.

Number of Average Oranges	Number of Partial Oranges	Number of Oranges Counted by Photo Interpreters
57	85	74

PART B MULTIVARIATE ANALYSIS

I. Introduction

Results from preliminary investigations using the discriminant analysis procedure in SAS have been very encouraging. The classification criterion was determined by a measure of generalized square distance. Three groups were used and the distance measure was based on the individual within-group covariance matrices since they were significantly different at the 0.001 level. The three groups which were used were sky, background (foilage, tree, etc.), and fruit. These groups were used since, when the entire photographs were scanned, small areas were also scanned which contained only background, sky, or fruit. These small scan areas were rectangular, so when fruit were scanned, some background points were included. SAS data sets were formed from each of these small scan areas and were stored on tape. However, in order for SAS to access these data sets, they had to be moved back to disk.

II. Sample Selection

Input to the discriminant procedure required a minimum of two observations per group for calibration data to form the within-group covariance matrices in order to develop the classification criteria. However, at least ten observations should be used to obtain a good classification criterion. Since the transmission values are fairly homogeneous each group, systematic sampling was used for selection of calibration data for sky and background. Realizing, of course, that a systematic sample yields a biased estimate of the covariance matrix. As mentioned earlier, when the small areas containing fruit were scanned, some background points were included in the rectangular area. In order to make sure the calibration data for fruit was actually fruit, two different may be that two filters would yield good enough classification results for counting fruit, thus reducing the amount of scanning and data to handle.

IV. Results

These initial results should not be viewed as conclusive, however, there have been several findings which appear quite promising and should be explored further. On orange trees with ripe fruit, it appears there are no substantial differences in classification results between using 1) all four filters, 2) the clear, red, and blue filters, or 3) the red, green, and blue filters. Using our definition of "correct" classification, all the above combinations yielded 100% "correct" classification. The three filter combinations classified more than 99% of the data points into the same groups. Thus, although there were a few discrepencies on a point-by-point basis, they were nominal. Table 5 shows how many points were classified into each group by the various filter combinations on a slide-by-slide basis for both oranges and peaches.

In Appendix B, Tables 6-53 contain the simple statistics; the variance and covariance matrices and the pairwise squared distance between groups for each of the filter combinations listed in Table 6. It can be seen from these tables that the mean vectors of the groups are quite different. The method of grouping did an excellent job of separating the data points for all four trees. However, the limitation of this approach is the fact that one does not know the exact origin of each data point. If the formulated definitions were correct, the multivariate technique shows a promising approach to automated fruit counting.

TABLE 5

Point-by-point classification results by slide, by type, by class, and by filter combination

Slide	Type	Filters	Foilage	Fruit	Sky
A	Orange	CRGB	622	222	50
	Orange	CRB	623	220	51
	Orange	RGB	620	224	50
B	Orange	CRGB	582	260	52
	Orange	CRB	574	268	52
	Orange	RGB	576	268	50
C	Peach	CRGB	328	287	51
	Peach'	CRB	332	282	52
	Peach	RGB	346	272	48
	Peach	CGB	358	256	52
	Peach	CRG	338	277	51
D	Peach	CRGB	348	230	286
	Peach	CRB	369	213	282
	Peach	CRG	376	202	286
	Peach	CGB	315	271	278
	Peach	RGB	356	213	286

The results for peaches are not as good as for oranges. The filter combinations containing the red filter showed nearly the same classification results with less than 2% of the points being classified into different groups. However, when the red filter was omitted, the classification results changes substantially, as can be seen from the previous table. Thus, it appears the red filter makes a substantial contribution for classification of peaches.

C. Recommendations

This study points out the need for further research in the following areas:

1. A test needs to be conducted using known fruit and various backgrounds particularly with the multivariate approach. That is, a sample of known fruit and backgrounds should be scanned. Then a sub-sample of these could be used as training and the remainder used in the test. This would permit one to evaluate how well one can classify fruit.
2. Determine the optimum magnification level, aperture size, and shape, and sampling interval for scanning.
3. Compare the results from the parametric multivariate classification model with a non-parametric classification model, for a suitable number of samples.
4. Tests should be made to determine if the mean vectors and covariance matrices are significantly different between fruit within each tree and between trees within the same maturity category.
5. Develop a sophisticated contour tracing algorithm which is capable of counting connected sets of points.
6. Develop an automated fruit counting system which could support an operational program both time and cost-wise.

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Appendix A

Theorem II: For any finite two dimensional real array $A = (a_{ij})$, if one successively applies the operation of eliminating runs less than length p in the horizontal direction, then eliminating runs less than length q in the vertical direction, and continues to alternate operating on the rows and columns until no further elimination is possible, then the resultant array $H = (h_{ij})$ is equivalent, in the sense that the elements not eliminated are in the same positions for all a_{ij}, to the array $V = (v_{ij})$ where V was obtained by first operating on the columns and then on the rows and so on until data reduction terminated. Furthermore, the smallest subset which will not be eliminated is of dimension $(p + 1) \times (q + 1)$.

Appendix B

Tables 6 - 53

Table 6--Simple statistics for orange tree A: Four variables - clear, red, green, and blue.

Variable	N	Sum	Mean	Variance	Standard Deviation
			Background		
Clear	38	12878.00	338.89	35058.09	187.23
Red	38	21584.00	568.00	53741.18	231.82
Green	38	16012.00	421.36	42642.40	206.50
Blue	38	18580.00	488.94	48269.34	219.70
			Fruit		
Clear	16	3300.00	206.25	738.60	27.17
Red	16	3142.00	196.37	505.71	22.48
Green	16	3690.00	230.62	715.05	26.74
Blue	16	8386.00	524.12	3531.45	59.42
			Sky		
Clear	10	310.00	31.00	39.33	6.27
Red	10	650.00	65.00	255.33	15.97
Green	10	774.00	77.40	127.15	11.27
Blue	10	900.00	90.00	136.88	11.69

Table 7--Variance and covariance matrices for orange tree A: four variables - clear, red, green, and blue.

Variable	Clear	Red	Green	Blue
		Background		
Clear	35058.09	43007.89	38378.31	40209.72
Red	43007.89	53741.18	46990.81	49732.21
Green	38378.31	46990.81	42642.40	43112.18
Blue	40209.72	49732.21	43112.18	48269.34
		Fruit		
Clear	738.60	550.56	692.10	1476.50
Red	550.56	505.71	554.41	917.81
Green	692.10	554.41	715.05	1245.78
Blue	1476.50	917.81	1245.78	3531.45
		Sky		
Clear	39.33	91.33	66.44	65.77
Red	91.33	255.33	170.00	158.66
Green	66.44	170.00	127.15	127.55
Blue	65.77	158.66	127.55	136.88

Table 8--Pairwise squared generalized distances between background, fruit, and sky[1/] for orange tree A.

Generalized Square Distance to Group			
From Group	Background	Fruit	Sky
Background	30.43	125.64	50.20
Fruit	1050.93	20.09	98.03
Sky	5677.12	18136.20	10.02

1/ where: $D^2\,(I|J) = (\bar{x}_I - \bar{x}_J)\ \mathrm{Cov}_J^{-1}\,(\bar{x}_I - \bar{x}_J) + \mathrm{LN}\,|\mathrm{Cov}_J|$

Table 9--Simple statistics for orange tree A: Three variables - clear, red, and blue.

Variables	N	Sum	Mean	Variance	Standard Deviation
			Background		
Clear	38	12878.00	338.89	35058.09	187.23
Red	38	21584.00	568.00	53741.18	231.82
Blue	38	18580.00	488.94	48269.34	219.70
			Fruit		
Clear	16	3300.00	206.25	738.60	27.17
Red	16	3142.00	196.37	505.71	22.48
Blue	16	8386.00	524.12	3531.45	59.42
			Sky		
Clear	10	310.00	31.00	39.33	6.27
Red	10	650.00	65.00	255.33	15.97
Blue	10	900.00	90.00	136.88	11.69

Table 10--Variance and covariance matrices for orange tree A : Three variables - clear, red, and blue.

Variables	Clear	Red	Blue
		Background	
Clear	35058.09	43007.89	40209.72
Red	43007.89	53741.18	49732.21
Blue	40209.72	49732.21	48269.34
		Fruit	
Clear	738.60	550.56	1476.50
Red	550.56	505.71	917.81
Blue	1476.50	917.81	3531.45
		Sky	
Clear	39.33	91.33	65.77
Red	91.33	255.33	158.66
Blue	65.77	158.66	136.88

Table 11--Pairwise squared generalized distances between background, fruit, and sky[1/] for orange tree A:

	Generalized Squared Distance to Group		
From Group	Background	Fruit	Sky
Background	24.94	107.63	43.67
Fruit	1037.30	16.59	89.85
Sky	3748.96	3773.38	10.70

1/ where: $D^2\ (I|J) = (\bar{x}_I - \bar{x}_J)'\ Cov^{-1}_J\ (\bar{x}_I - \bar{x}_J) + LN\ |Cov_J|$

Table 12--Simple statistics for orange tree A : Three variables - red, green, and blue.

Variable	N	Sum	Mean	Variance	Standard Deviation
			Background		
Red	38	21584.00	568.00	53741.18	231.82
Green	38	16012.00	421.36	42642.40	206.50
Blue	38	18580.00	488.94	48269.34	219.70
			Fruit		
Red	16	3142.00	196.37	505.71	22.48
Green	16	3690.00	230.62	715.05	26.74
Blue	16	8386.00	524.12	3531.45	59.42
			Sky		
Red	10	650.00	65.00	255.33	15.97
Green	10	774.00	77.40	127.15	11.27
Blue	10	900.00	90.00	136.88	11.69

Table 13--Variance and covariance matrices for orange tree A: Three variables - red, green, and blue.

Variables	Red	Green	Blue
		Background	
Red	53741.18	46990.81	49732.21
Green	46990.81	42642.40	43112.18
Blue	49732.21	43112.18	48269.34
		Fruit	
Red	505.71	554.41	917.81
Green	554.41	715.05	1245.78
Blue	917.81	1245.78	3531.45
		Sky	
Red	255.33	170.00	158.66
Green	170.00	127.15	127.55
Blue	158.66	127.55	136.88

Table 14--Pairwise squared generalized distances between background, fruit, and sky[1] for orange tree A :

Generalized Squared Distance to Group

From Group	Background	Fruit	Sky
Background	25.91	118.49	40.25
Fruit	767.17	18.09	76.06
Sky	2337.14	16281.54	9.54

1/ where: $D^2\ (I|J) = (\bar{x}_I - \bar{x}_J)'\ Cov_J^{-1}\ (\bar{x}_I - \bar{x}_J) + LN\ |Cov_J|$

Table 15--Simple statistics for orange tree B: Four variables - red, green, blue, and clear.

Variable	N	Sum	Mean	Variance	Standard Deviation
			Background		
Red	32	18694.00	584.18	46314.93	215.20
Green	32	15358.00	479.93	40154.44	200.38
Blue	32	18412.00	575.37	42647.59	206.51
Clear	32	12990.00	405.93	34587.99	185.97
			Fruit		
Red	16	1728.00	108.00	786.66	28.04
Green	16	3448.00	215.50	1254.66	35.42
Blue	16	6946.00	434.12	2525.58	50.25
Clear	16	3020.00	188.75	1096.46	33.11
			Sky		
Red	10	650.00	65.00	255.33	15.97
Green	10	774.00	77.40	127.15	11.27
Blue	10	900.00	90.00	136.88	11.69
Clear	10	310.00	31.00	39.33	6.27

Table 16--Variance and covariance matrices for orange tree B: Four variables - red, green, blue, and clear.

Variable	Red	Green	Blue	Clear
		Background		
Red	46314.93	42816.14	43386.95	39776.91
Green	42816.14	40154.44	40282.28	37084.64
Blue	43386.95	40282.28	42647.59	38092.50
Clear	39776.91	37084.64	38029.50	34587.99
		Fruit		
Red	786.66	909.60	1188.26	847.20
Green	909.60	1254.66	1580.60	1151.60
Blue	1188.26	1580.60	2525.58	1582.03
Clear	847.20	1151.60	1582.03	1096.46
		Sky		
Red	255.33	170.00	158.66	91.33
Green	170.00	127.15	127.55	66.44
Blue	158.66	127.55	136.88	65.77
Clear	91.33	66.44	65.77	39.33

Table 17 --Pairwise squared generalized distances between background, fruit, and sky[1] for orange tree B.

Generalized Squared Distance to Group

From Group	Background	Fruit	Sky
Background	27.82	122.61	100.75
Fruit	910.05	20.20	142.96
Sky	8734.00	8209.17	11.02

1/

where: $D^2(I|J) = (\bar{x}_I - \bar{x}_J)' \, Cov_J^{-1} \, (\bar{x}_I - \bar{x}_J) + LN \, |Cov_J|$

Table 18-- Variance and covariance matrices for orange tree B: Three variables - clear, red, and blue.

Variables	Clear	Red	Blue
		Background	
Clear	34587.99	39776.91	38029.50
Red	39776.91	46314.93	43386.95
Blue	38029.50	43386.95	42647.59
		Fruit	
Clear	1096.46	847.20	1582.03
Red	847.20	786.66	1188.26
Blue	1582.03	1188.26	2525.58
		Sky	
Clear	39.33	91.33	65.77
Red	91.33	255.33	158.66
Blue	65.77	158.66	136.88

Table 19--Simple statistics for orange tree B: Three variables - clear, red, and blue.

Variable	N	Sum	Mean	Variance	Standard Deviation
			Background		
Clear	32	12990.00	405.93	34587.99	185.97
Red	32	18694.00	584.18	46314.93	215.20
Blue	32	18412.00	575.37	42647.59	206.51
			Fruit		
Clear	16	3020.00	188.75	1096.46	33.11
Red	16	1728.00	108.00	786.66	28.04
Blue	16	6946.00	434.12	2525.58	50.25
			Sky		
Clear	10	310.00	31.00	39.33	6.27
Red	10	650.00	65.00	255.33	15.97
Blue	10	900.00	90.00	136.88	11.69

Table 20--Pairwise squared generalized distances between background, fruit, and sky[1/] for orange tree B .

Generalized Squared Distance to Group

From Group	Background	Fruit	Sky
Background	23.23	117.05	66.69
Fruit	816.93	17.33	126.63
Sky	6775.07	3655.53	10.70

1/

where: $D^2 (I|J) = (\bar{x}_I - \bar{x}_J) \; Cov_J^{-1} \; (\bar{x}_I - \bar{x}_J) + LN \; |Cov_J|$

Table 21--Simple statistics for orange tree B: Three variables - red, green, and blue.

Variable	N	Sum	Mean	Variance	Standard Deviation
			Background		
Red	32	18694.00	584.18	46314.93	215.20
Green	32	15358.00	479.93	40154.44	200.38
Blue	32	18412.00	575.37	42647.59	206.51
			Fruit		
Red	16	1728.00	108.00	786.66	28.04
Green	16	3448.00	215.50	1254.66	35.42
Blue	16	6946.00	434.12	2525.58	50.25
			Sky		
Red	10	650.00	65.00	255.33	15.97
Green	10	774.00	77.40	127.15	11.27
Blue	10	900.00	90.00	136.88	11.69

Table 22-- Variance and covariance matrices for orange tree B : Three variables - red, green, and blue.

Variables	Red	Green	Blue
		Background	
Red	46314.93	42816.14	43386.95
Greed	42816.14	40154.44	40282.28
Blue	43386.95	40282.28	42647.59
		Fruit	
Red	786.66	909.60	1188.26
Green	909.60	1254.66	1580.60
Blue	1188.26	1580.60	2525.58
		Sky	
Red	255.33	170.00	158.66
Green	170.00	127.15	127.55
Blue	158.66	127.55	136.88

Table 23--Pairwise squared generalized distances between background, fruit, and sky[1/] for orange tree B .

	Generalized Squared Distance to Group		
From Group	Background	Fruit	Sky
Background	24.66	115.99	41.21
Fruit	867.94	18.23	127.86
Sky	2679.48	6300.20	9.54

1/

Where: $D^2\ (I|J) = (\bar{x}_I - \bar{x}_J)'\ Cov_J^{-1}\ (\bar{x}_I - \bar{x}_J) + LN\ |Cov_J|$

Table 24--Simple statistics for peach tree C: Four variables - clear, red, green, and blue.

Variable	N	Sum	Mean	Variance	Standard Deviation
			Background		
Clear	25	8438.00	337.52	16407.42	128.09
Red	25	14030.00	561.20	28003.33	167.34
Green	25	11380.00	455.20	18591.33	136.35
Blue	25	12328.00	493.12	19055.69	138.04
			Fruit		
Clear	20	3556.00	177.80	10963.32	104.70
Red	20	4798.00	239.90	22577.04	150.25
Green	20	5150.00	257.50	11912.36	109.14
Blue	20	7312.00	365.60	14230.98	119.29
			Sky		
Clear	10	310.00	31.00	39.33	6.27
Red	10	650.00	65.00	255.33	15.97
Green	10	774.00	77.40	127.15	11.27
Blue	10	900.00	90.00	136.88	11.69

Table 25--Variance and covariance matrices for peach tree C : Four variables - clear, red, green, and blue.

Variable	Clear	Red	Green	Blue
		Background		
Clear	16407.42	21324.43	17416.76	17615.56
Red	21324.43	28003.33	22675.83	22904.26
Green	17416.76	22675.83	18591.33	18592.60
Blue	17615.56	22904.26	18592.60	19055.69
		Fruit		
Clear	10963.32	15542.08	11391.26	12367.07
Red	15542.08	22577.04	16225.63	17382.69
Green	11391.26	16225.63	11912.36	12843.15
Blue	12367.07	17382.69	12843.15	14230.98
		Sky		
Clear	39.33	91.33	66.44	65.77
Red	91.33	255.33	170.00	158.66
Green	66.44	170.00	127.15	127.55
Blue	65.77	158.66	127.55	136.88

Table 26--Pairwise squared generalized distances between background, fruit and sky[1/] for peach tree C .

Generalized Squared Distance to Group

From Group	Background	Fruit	Sky
Background	23.10	120.11	630.08
Fruit	53.22	25.24	82.33
Sky	3951.24	2272.05	11.02

1/

where: $D^2\ (I|J) = (\bar{x}_I - \bar{x}_J)'\ Cov_J^{-1}\ (\bar{x}_I - \bar{x}_J) + LN\ |Cov_J|$

Table 27--Simple statistics for peach tree C: Three variables - clear, red, and blue.

Variable	N	Sum	Mean	Variance	Standard Deviation
			Background		
Clear	25	8438.00	337.52	16407.42	128.09
Red	25	14030.00	561.20	28003.33	167.34
Blue	25	12328.00	493.12	19055.69	138.04
			Fruit		
Clear	20	3556.00	177.80	10963.32	104.70
Red	20	4798.00	239.90	22577.04	150.25
Blue	20	7312.00	365.60	14230.98	119.29
			Sky		
Clear	10	310.00	31.00	39.33	6.27
Red	10	650.00	65.00	255.33	15.97
Blue	10	900.00	90.00	136.88	11.69

Table 28--Variance and covariance matrices for peach tree C: Three variables - clear, red, and blue.

Variable	Clear	Red	Blue
	Background		
Clear	16407.42	21324.43	17615.56
Red	21324.43	28003.33	22904.26
Blue	17615.56	22904.26	19055.69
	Fruit		
Clear	10963.32	15542.08	12367.07
Red	15542.08	22577.04	17382.69
Blue	12367.07	17382.69	14230.98
	Sky		
Clear	39.33	91.33	65.77
Red	91.33	255.33	158.66
Blue	65.77	158.66	136.88

Table 29--Pairwise squared generalized distances between background, fruit and sky[1] for peach tree C .

Generalized Squared Distance to Group

From Group	Background	Fruit	Sky
Background	20.33	82.71	94.32
Fruit	42.90	21.07	67.58
Sky	3718.57	1302.59	10.70

1/

where: $D^2\ (I|J) = (\overline{x}_I - \overline{x}_J)'\ \mathrm{Cov}_J^{-1}\ (\overline{x}_I - \overline{x}_J) + \mathrm{LN}\ |\mathrm{Cov}_J|$

Table 30--Simple statistics for peach tree C: Three variables - red, green, and blue.

Variable	N	Sum	Mean	Variance	Standard Deviation
			Background		
Red	25	14030.00	561.20	28003.33	167.34
Green	25	11380.00	455.20	18591.33	136.35
Blue	25	12328.00	493.12	19055.69	138.04
			Fruit		
Red	20	4798.00	239.90	22577.04	150.25
Green	20	5150.00	257.50	11912.36	109.14
Blue	20	7312.00	365.60	14230.98	119.29
			Sky		
Red	10	650.00	65.00	255.33	15.97
Green	10	774.00	77.40	127.15	11.27
Blue	10	900.00	90.00	136.88	11.69

Table 31--Variance and covariance matrices for peach tree C: Three variables - red, green, and blue.

Variable	Red	Green	Blue
	Background		
Red	28003.33	22675.83	22904.26
Green	22675.83	18591.33	18592.60
Blue	22904.26	18592.60	19055.69
	Fruit		
Red	22577.04	16225.63	17382.69
Green	16225.63	11912.36	12843.15
Blue	17382.69	12843.15	14230.98
	Sky		
Red	255.33	170.00	158.66
Green	170.00	127.15	127.55
Blue	158.66	127.55	136.88

Table 34---Variance and covariance matrices for peach tree C : Three variables - clear, green, and blue.

Variable	Clear	Green	Blue
	Background		
Clear	16407.42	17416.76	17615.56
Green	17416.76	18591.33	18592.60
Blue	17615.56	18592.60	19055.69
	Fruit		
Clear	10963.32	11391.26	12367.07
Green	11391.26	11912.36	12843.15
Blue	12367.07	12843.15	14230.98
	Sky		
Clear	39.33	66.44	65.77
Green	66.44	127.15	127.55
Blue	65.77	127.55	136.88

Table 35--Pairwise squared generalized distances between background, fruit, and sky[1/] for peach tree C.

Generalized Squared Distance to Group			
From Group	Background	Fruit	Sky
Background	17.83	33.79	551.23
Fruit	43.67	19.27	76.32
Sky	3940.75	2121.60	8.54

1/1

where: $D^2(I|J) = (\bar{x}_I - \bar{x}_J)' \, \mathrm{Cov}_J^{-1} (\bar{x}_I - \bar{x}_J) + \mathrm{LN}\, |\mathrm{Cov}_J|$

Table 36--Simple statistics for **peach** tree C: Three variables - clear, red, and green.

Variable	N	Sum	Mean	Variance	Standard Deviation
			Background		
Clear	25	8438.00	337.52	16407.42	128.09
Red	25	14030.00	561.20	18003.33	167.34
Green	25	11380.00	455.20	18591.33	136.35
			Fruit		
Clear	20	3556.00	177.80	10963.32	104.70
Red	20	4798.00	239.90	22577.04	150.25
Green	20	5150.00	257.50	11912.36	109.14
			Sky		
Clear	10	310.00	31.00	39.33	6.27
Red	10	650.00	65.00	255.33	15.97
Green	10	774.00	77.40	127.15	11.27

Table 37--Variance and covariance matrices for peach tree C: Three variables - clear, red, and green.

Variable	Clear	Red	Green
	Background		
Clear	16407.42	21324.43	17416.76
Red	21324.43	28003.33	22675.83
Green	17416.76	22675.83	18591.33
	Fruit		
Clear	10963.32	15542.08	11391.26
Red	15542.08	22577.04	16225.63
Green	11391.26	16225.63	11912.36
	Sky		
Clear	39.33	91.33	66.44
Red	91.33	255.33	170.00
Green	66.44	170.00	127.15

Table 38--Pairwise squared generalized distances between background, fruit, and sky[1/] for peach tree C .

Generalized Squared Distance to Group			
From Group	Background	Fruit	Sky
Background	19.95	67.96	74.42
Fruit	43.78	19.78	39.64
Sky	3885.89	1200.67	9.65

1/ where: $D^2(I|J) = (\bar{x}_I - \bar{x}_J)' \mathrm{Cov}_J^{-1} (\bar{x}_I - \bar{x}_J) + \mathrm{LN}\,|\mathrm{Cov}_J|$

Table 39--Simple statistics for peach tree D: Four variables - clear, red, green, and blue.

Variable	N	Sum	Mean	Variance	Standard Deviation
			Background		
Clear	22	10836.00	492.54	37287.30	193.09
Red	22	14600.00	663.63	76052.43	275.77
Green	22	11966.00	543.90	47011.22	216.82
Blue	22	14878.00	676.27	40298.39	200.74
			Fruit		
Clear	17	6208.00	365.17	31003.02	176.07
Red	17	5818.00	342.23	72116.94	268.54
Green	17	7688.00	452.23	36840.94	191.93
Blue	17	8648.00	508.70	34673.47	186.20
			Sky		
Clear	22	2858.00	129.90	19.61	4.42
Red	22	8940.00	406.36	132.43	11.50
Green	22	5690.00	258.63	30.24	5.49
Blue	22	3406.00	154.81	25.77	5.07

Table 40--Variance and covariance matrices for peach tree D : Four variables - clear, red, green, and blue.

Variable	Clear	Red	Green	Blue
		Background		
Clear	37287.30	51023.25	41721.19	38137.74
Red	51023.25	76052.43	57155.01	51657.81
Green	41721.19	57155.01	47011.22	42089.93
Blue	38137.74	51657.81	42089.93	40298.39
		Fruit		
Clear	31003.02	45007.45	33351.45	31819.86
Red	45007.45	72116.94	46615.94	47594.07
Green	33351.45	46615.94	36840.94	32994.57
Blue	31819.86	47594.07	32994.57	34673.47
		Sky		
Clear	19.61	38.12	17.96	20.26
Red	38.12	132.43	37.66	46.35
Green	17.96	37.66	30.24	18.88
Blue	20.26	46.35	18.88	25.77

Table 41--Pairwise squared generalized distances between background, fruit, and sky[1] for peach tree D.

Generalized Squared Distance to Group

From Group	Background	Fruit	Sky
Background	30.49	50.53	111.56
Fruit	34.32	30.58	169.30
Sky	20208.35	14974.17	11.04

1/

where: $D^2\ (I|J) = (\overline{x}_I - \overline{x}_J)'\ Cov_J^{-1}\ (\overline{x}_I - \overline{x}_J) + Ln\ |Cov_J|$

Table 42---Simple statistics for peach tree D: Three variables - clear, red, and blue.

Variable	N	Sum	Mean	Variance	Standard Deviation
			Background		
Clear	22	10836.00	492.54	37287.30	193.09
Red	22	14600.00	663.63	76052.43	275.77
Blue	22	14878.00	676.27	40298.39	200.74
			Fruit		
Clear	17	6208.00	365.17	31003.02	176.07
Red	17	5818.00	342.23	72116.94	268.54
Blue	17	8648.00	508.70	34673.47	186.20
			Sky		
Clear	22	2858.00	129.90	19.61	4.42
Red	22	8940.00	406.36	132.43	11.50
Blue	22	3406.00	154.81	25.77	5.07

Table 43--Variance and covariance matrices for peach tree D: Three variables - clear, red, and blue.

Variables	Clear	Red	Blue
		Background	
Clear	37287.30	51023.25	38137.74
Red	51023.25	76052.43	51657.81
Blue	38137.74	51657.81	40298.39
		Fruit	
Clear	31003.02	45007.45	31819.86
Red	45007.45	72116.94	47594.07
Blue	31819.86	47594.07	34673.47
		Sky	
Clear	19.61	38.12	20.26
Red	38.12	132.43	46.35
Blue	20.26	46.35	25.77

Table 44--Pairwise squared generalized distances between background, fruit, and sky[1/] for peach tree D.

	Generalized Squared Distance to Group		
From Group	Background	Fruit	Sky
Background	26.39	32.28	52.69
Fruit	29.93	26.61	74.99
Sky	20158.34	14970.92	8.42

1/ where: $D^2 (I|J) = (\bar{x}_I - \bar{x}_J)' \, Cov_J^{-1} \, (\bar{x}_I - \bar{x}_J) + LN \, |Cov_J|$

Table 45--Simple statistics for peach tree D: Three variables - clear, red, and green.

Variable	N	Sum	Mean	Variance	Standard Deviation
			Background		
Clear	22	10836.00	492.54	37287.30	193.09
Red	22	14600.-0	663.63	76052.43	275.77
Green	22	11966.00	543.90	47011.22	216.82
			Fruit		
Clear	17	6208.00	365.17	31003.02	176.07
Red	17	5818.00	342.23	72116.94	268.54
Green	17	7688.00	452.23	36840.94	191.93
			Sky		
Clear	22	2858.00	129.90	19.61	4.42
Red	22	8940.00	406.36	132.43	11.50
Green	22	5690.00	258.63	30.24	5.49

Table 46--Variance and covariance matrices for peach tree D : Three variables - clear, red, and green.

Variables	Clear	Red	Green
		Background	
Clear	37287.30	51023.25	41721.19
Red	51023.25	76052.43	57155.01
Green	41721.19	57155.01	47011.22
		Fruit	
Clear	31003.02	45007.45	33351.45
Red	45007.45	72116.94	46615.94
Green	33351.45	46615.94	36840.94
		Sky	
Clear	19.61	38.12	17.96
Red	38.12	132.43	37.66
Green	17.96	37.66	30.24

Table 47--Pairwise squared generalized distances between background, fruit, and sky[1/] for peach tree D.

Generalized Squared Distance to Group

From Group	Background	Fruit	Sky
Background	25.05	37.31	80.17
Fruit	28.78	25.34	109.10
Sky	10205.19	7499.31	9.65

1/

where: $D^2\ (I|J) = (\bar{x}_I - \bar{x}_J)'\ Cov^{-1}_J\ (\bar{x}_I - \bar{x}_J) + LN\ |Cov_J|$

Table 48--Simple statistics for peach tree D: Three variables - clear, green, and blue.

Variable	N	Sum	Mean	Variance	Standard Deviation
			Background		
Clear	22	10836.00	492.54	37287.30	193.09
Green	22	11966.00	543.90	47011.22	216.82
Blue	222	14878.00	676.27	40298.39	200.74
			Fruit		
Clear	17	6208.00	365.17	31003.02	176.07
Green	17	7688.00	452.23	36840.94	191.93
Blue	17	8648.00	508.70	34673.47	186.20
			Sky		
Clear	22	2858.00	129.90	19.61	4.42
Green	22	5690.00	258.63	30.24	5.49
Blue	22	3406.00	154.81	25.77	~~7~~5.07

Table 49--Variance and covariance matrices for peach tree D : Three variables - clear, green, and blue.

Variables	Clear	Green	Blue
		Background	
Clear	37287.30	41721.19	38137.74
Green	41721.19	47001.22	42089.93
Blue	38137.74	42089.93	40298.39
		Fruit	
Clear	31003.02	33351.45	31819.86
Green	33351.45	36840.94	32994.57
Blue	31819.86	32994.57	34673.47
		Sky	
Clear	19.61	17.96	20.26
Green	17.96	30.24	18.88
Blue	20.26	18.88	25.77

Table 50--Pairwise squared generalized distances between background, fruit, and sky[1] for peach tree D.

Generalized Squared Distance to Group			
From Group	Background	Fruit	Sky
Background	21.87	40.97	84.89
Fruit	26.99	23.27	31.76
Sky	11401.07	5431.71	7.17

1/

where: $D^2\ (I|J) = (\bar{x}_I - \bar{x}_J)'\ Cov_J^{-1}\ (\bar{x}_I - \bar{x}_J) + LN\ |Cov_J|$

Table 51--Simple statistics for **peach** tree D: Three variables - red, green, and blue.

Variable	N	Sum	Mean	Variance	Standard Deviation
			Background		
Red	22	14600.00 00	663.63	76052.43	275.77
Green	22	11966.00	543.90	47011.22	216.82
Blue	22	14878.00	676.27	40298.39	200.74
			Fruit		
Red	17	5818.00	342.23	72116.94	268.54
Green	17	7688.00	452.23	36840.94	191.93
Blue	17	8648.00	508.70	34673.47	186.20
			Sky		
Red	22	8940.00	406.36	132.43	11.50
Green	22	5690.00	258.63	30.24	5.49
Blue	22	3406.00	154.81	25.77	5.07

Table 52--Variance and covariance matrices for peach tree D : Three variables - red, green, and blue.

Variables	Red	Green	Blue
	Background		
Red	76052.43	57155.01	51657.81
Green	57155.01	47011.22	42089.93
Blue	51657.81	42089.93	40298.39
	Fruit		
Red	72116.94	46615.94	47594.07
Green	46615.94	36840.94	32994.57
Blue	47594.07	32994.57	34673.47
	Sky		
Red	132.43	37.66	46.35
Green	37.66	30.24	18.88
Blue	46.35	18.88	25.77

Table 53--Pairwise squared generalized distances between background, fruit, and sky[1] for peach tree D .

	Generalized Squared Distance to Group		
From Group	Background	Fruit	Sky
Background	27.40	36.19	59.18
Fruit	31.28	27.83	76.23
Sky	20126.45	14882.12	9.92

1/

where: $D^2 (I|J) = (\bar{x}_I - \bar{x}_J)' \, Cov_J^{-1} \, (\bar{x}_I - \bar{x}_J) + LN \, |Cov_J|$