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Spatial Scale of Crop-Yield Models

A Review of the Relationship Between Scale of Models and Accuracy

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Abstract

The final usefulness of the physiologically based weather-crop yield models now being developed will depend in part upon the degree to which point observations of weather phenomena can be extended to surrounding regions.

This report reviews methods of estimating areal data from point estimates, methods of relating macro and micro climatic data, and discusses the conflicts involved in treating such problems as accuracy vs. scale, correlation decay, and limits of scale interpolation. Two areas of investigation that could be pursued with respect to these problems would involve the use of either "synoptic climatology" or of data interpolation methods. Of these two methods, a synoptic classification procedure seems more desirable.

Key words: Yield models, synoptic climatology, data interpolation, scale.

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Spatial Scale of Crop-Yield Models:
A review of the Relationship between Scale of Models and Accuracy

Bruce w. Strand

Introduction

The U.S. Dept. of Agriculture has started to develop and evaluate complex, physiologically based weather-crop yield models to meet its need to generate ever-more accurate estimates of grain yields and production. The implementation of such models, designed for small areas, for the construction of yield estimates for large areas, would require effective use of procedures for generalizing point weather observations to large areas.

This paper, the final report of a three month literature search for the Yield Research Branch, Statistical Research Division, Economics and Statistics Service, U.S. Dept. of Agriculture, reviews methods of applying small scale models to large areas, notes limitations to those methods, and identifies research that should be considered in any selection or evaluation of models.

Weather-crop yield Models

In general, area weather-crop yield equations relate final yields, or in-season crop growth and development to weather and soils data through solving a series of simultaneous equations. Five types of statistical studies can be used to estimate weather-crop yield models. They are:

- 1) Analysis of long series of yield data from one particular site(i.e., time series)
- 2) Analysis of yield data from many sites during one particular year (i.e., cross-sectional studies)
- 3) A combination of 1) and 2)
- 4) Studies of the relationship of yield forming processes to weather conditions during short periods of time.
- 5) Studies in which the soil is standardized to eliminate spatial and/or edaphic differences [36]

Several yield models of interest to U.S.D.A. have been produced by methods 1) and 4) above. The yield models of Arkin, et.al. [1,2] are examples of type #4). Studies that consider, contain or use spatial information are of types 2) and 5). The 'Thompson' type yield models are an example of type #1)[42].

Weather-crop yield models could exist on the same continuum of scale as their input data. However, it is necessary to divide up the atmospheric as well as the soil, plant and water phenomena into distinct classes for the purpose of model application. While disagreements as to class intervals exist, the following scales and their limits are often found in meteorology[15]:

- Micro-scale .01 to 100 M
- Local scale 100 to 5,000 M
- Meso-scale 1000 to 20,000 M
- Macro-scale 10,000 to 10,000,000 M

Weather-crop yield models generally are designed to estimate final crop yields at the same scale as their input data. State crop yields are estimated with equations using statewide weather averages. Weather-crop yield models for entire countries are generally yearly models, state level yield models tend to be monthly models, and plot scale models may even require daily and hourly data. Most models have been developed using either large area or plot data. Meso-scale models, (i.e., weekly models, 10-500 SQ.KM.), have seldom been developed. This is due to the unavailability of weather and crop data at these scales. Figure #1 below diagrams this relationship[39].

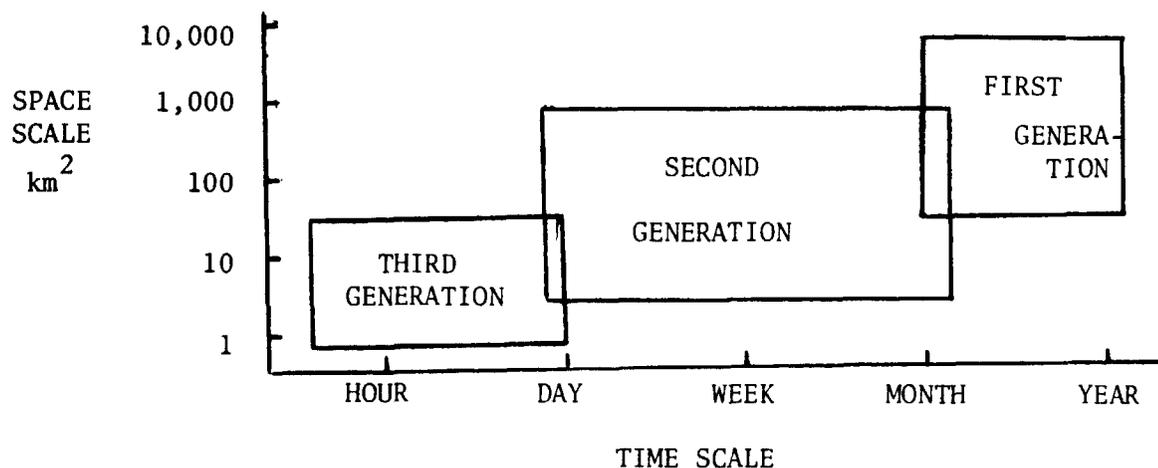


Figure #1-Yield models vs. time and space inputs

Obviously, the scale of the input data of the model has strong control over the scale of the model. Unfortunately, it is seldom that all the data needed for a model are available at the same spatial and temporal scale. Crop yield data may be available at the county level, whereas weather data may only be available for larger regions. Also, weather data may be available daily, but crop condition or growth stage data may only be available for a few times during the growing season, if at all.

Small scale plot developed ('third generation' in figure #1) weather-crop yield models are being considered for use in yield production estimation by the U.S. Department of Agriculture because they appear to offer increased precision in estimating final crop yields in contrast to large area models. Their increased precision is due to several factors. Because these models were designed to operate in a manner similar to actual plant processes, they are better able to 'describe' plant growth and development processes than regression models that relate weather variables to final yields. Secondly, detailed weather and crop information is used to create the model (i.e., estimate model coefficients). These detailed weather and crop measurements are more likely to represent the actual growing conditions at a place than are large area estimates. Thirdly, by only considering a small region, these models are not likely to mix input variables which have different scales. Evaluation of the suitability of these models for large area production

estimates involves determining the relationship between model performance and spatial scale. Because any small scale weather-crop yield model will need to be applied over the entire crop growing region, a review of the relationship of models to scale, their accuracy and potential method of applying them over the entire region is necessary in order to choose the most appropriate scale of application. Satisfactory relationships between macro- and micro- scale yield models should stand between the extremes of over-simplification on the one hand and excessive refinement on the other. Oversimplification is likely to be seriously misleading because of the associated failure to recognize important variables.

Estimating Areal Data

There are two basic methods for estimating areal averages. One method is to divide the entire area into regions. This is usually done by fitting polygons around or between the data points. For example, if weather data are to be estimated for a large area, each weather station would be in the center of a polygon and the entire area would be divided into a series of polygons. Each weather station is then assigned a weight proportional to the (cropland) area of its polygon. Another method used by a commercial firm for economic modeling places weather stations at the vertices of triangles .

The second method of estimating areal data averages involves fitting a 'surface' to the area through the data points. If a least squares regression method is used to fit equations to the spatial data, the resulting equations are often called a 'trend surface'. The regression equation can be either a polynomial or a Fourier equation. This method was used by Runge to interpolate values between the closest weather stations[31]. His equation is:

$$W = (1/4\pi k) \exp(-r^2/4k),$$

where k is a parameter determining the shape of the weight factor which is to be related to the density of the observed data, and r is the distance. In contrast to this method, Feyerherm related plot based estimates of yield to crop reporting districts (CRD's) yields with linear regressions[10]. His final regression was:

$$CRD\{yield\} = 11.7 + 0.01 * (Plot\text{-}based\ yields)^2$$

indicating a curvilinear relationship between plot yields and CRD yields. Feyerherm suggested this relationship to account for 'grazed out' wheat losses in poor crop years. In addition, Feyerherm's equation does not consider the effect of the varying sizes of the CRD's on his regression equation.

The third method of relating micro- and macro- models is based on physical model building techniques. Civil engineering studies have attempted to resolve this problem by 'linearizing' the effect of spatial differences[30].

Either of these methods has shortcomings. In order to apply a plot developed yield model to a large region, an assumption must be made concerning the general relationships between weather and crop development and growth rates over several order magnitudes of scale. That is to say, the same coefficients found to be useful in relating, for example, locally measured solar radiation to locally determined crop growth rates should apply equally well for a regional average of solar radiation when related to a regional average for growth rate. If the relationships hold, then the relationships are linear over the several orders of magnitude. Such linear relationships may not be true. Feyerherm found that he had to use a non-linear relationship between yield at the plot level and the crop reporting district (CRD).

Aggregating or 'scaling up' a model may be as large a source of errors and variation as 'scaling down' to plot size. When observations are aggregated by areal units and recorded as totals, the variations within each unit is 'averaged out'. This averaging process removes surface features whose wavelengths are less than the size of the data unit [19]. Levels of aggregation of data should be related to the sensitivity scale of the variables under investigation. Such a procedure has not been applied to crop yield models.

Accuracy vs. Scale

Crop yield models derived from test plot data may show a marked reduction in accuracy when applied to large area estimates. The model input variables and their weights derived from plot size data may not be valid for large areas. Relationships between the micro-climate of a research plot and the locally measured weather variables may be more direct than the relationship of the large scale (synoptic scale) air movements to the local weather measurements. In fact, some of the variables useful in modeling crop yields at the plot level, i.e., water holding capacity per soil depth or soil textures per depth, may have little or no meaning at such larger spatial scales as NOAA Climatic Divisions. Local variations in these variables may be 'averaged out' with aggregation to larger areas.

In contrast, crop yield models can be applied to too small of areas with a different set of problems. First, there may be redundant information at very small scales. If the same general crop forming elements are found over an area appreciably larger than the scale at which the model was estimated, needless computations will occur. In such cases, one observation would probably be sufficient for the entire region. Secondly, because weather data are seldom collected at very fine grid levels, some data will need to be estimated via any of several methods.

Yield models use several input variables that may not have equal variance over changing spatial and temporal scales. Some variables such as winter minimum temperature may be fairly uniform over large areas or at least under the same synoptic patterns. Other variables such as summer

rainfall may need very dense gauge networks to achieve measurement accuracy for use in these models. In a study evaluating rain gauge network densities, Huff and Shipp found that there would need to be rain gauges every 2 miles in the warm seasons compared with 6 miles in the cold season to explain 90% of the variation in rainfall[17]. Huff found summertime convective rainfall to be 3 to 4 times more variable spatially than winter synoptic scale rainfall[16].

Rainfall Variability

Almost all crop yield models use rainfall data in their models. Rainfall tends to be the variable in the yield equations with the highest spatial and temporal variability. In marginal crop growing areas, moisture stress is often present and it is the single most important climatic variable controlling yield[44]. For some climates, higher variations in rainfall amounts occur more often during the summer months than in the winter. For example, summertime convective rainfall may be 2 to 3 times more variable than winter synoptic rainfall [16]. Variations over time in rainfall has been treated by numerous mathematical and statistical methods. Crutcher [9] found that time series rainfall could be fitted to a GAMMA density function, and has developed a computer program that estimates probabilities of rain.

Correlation decay is the rate at which the correlation between two observations decreases with increasing distances. Correlation decay formulas have the general form:

$$r(x,y)=e^{-b(x^2+y^2)^{1/2}},$$

where the correlation between a measurement at x and y are decreasingly related to each other by a rate b . In this equation, $(x^2+y^2)^{1/2}$ is the cartesian equation for distance between any two points, x and y. Rainfall patterns that fit this equation are called 'isotropic' patterns, meaning the correlation decay is equal in both the x and y direction.

Rainfall in some storms appears to be isotropic [33], but anisotropic rainfall correlations appear to be more common in the literature[14]. Sneva and Calvin [36] found that bearing affected correlation decay with minimum decay along the direction of the storm track. Rainfall amounts can be stratified by the type of storm that produce rain. Such stratification often reduces rainfall variability appreciably. The value of the exponent 'b' in the correlation decay equation has been shown to vary based on the synoptic type of the rainstorm. Therefore, any equation that seeks to estimate rainfall could contain an interaction term relating season and rainfall type . Huff and Ship [18],divided storms into three general synoptic types; frontal storms, air mass storms and low pressure center passage, and were able to substantially reduce rainfall variances. Patrinos and others [27], stratified rainfall records into 'wet' and 'dry' seasons, and achieved similar reductions in rainfall variability within weather 'type'. Huff and Shipp [16] found correlation decay was greatest in storms associated with thunderstorms and air mass storms.

Interaction between scale and Time

In general, little work has been done on the role of spatial information in yield models and interaction terms relating the joint effect of location with time or other important model variables are seldom used in yield equations. In that respect, most yield models can be considered to be 'one-dimensional', i.e., they only consider the time dimension. Stanhill [37] cites an Indian study which found "the interactions between sites and years was very large [having] a greater effect on yield than that of either year or site alone". Despite the possible importance of interactions between spatial and temporal variables in large area yield modeling, spatial interaction terms are lacking in most detailed yield models. More importantly, locally derived plot yield models could be used to estimate large area yield with an undetected but large error in estimate. For example, solar radiation (R_s) is measured at only 22 sites throughout the Great Plains. If R_s is to be used in yield equations for the whole Great Plains region, it will need to be estimated for the region using these 22 observation points. While such a program is underway, this density may not be sufficient as a basis for estimating R_s during certain critical seasons of the year[41]. They consider only variations over time rather than space.

Covariance models using time and space information appear on the literature but are not well known. One model proposed by Smit[35] to include time and space covariance represents a number of time slices, each slice describing spatial relationships that are assumed to be constant over the N spatial units for the time point. From this a comparison of parameters among the time slices considers the stability of the spatial relationships over the T time points. In one study, Sakamoto, Strommen and LeDuc investigated the result of differing densities of weather data on a large area, 'first generation' crop yield model and found no significant differences in the performance of the model using the more dense network[32]. In a similar study, Greene and others [12] using state-wide CCEA model and the smaller scale CRD models for Oklahoma wheat yield for 1931-1973 also found no advantage to using the smaller scale CRD yield models over the state wide yield models. These results could be due to several reasons. For one, the researchers estimated weather data from larger scale maps and inferred this to the crop district scale based on isohyetal maps. While the data came from two sources, they both estimated to the same spatial scale. Thus the models should show similar results. Secondly, by averaging weather data for large areas such as CRD's, the tendency is to depress the extreme conditions that might otherwise be present if a smaller unit was analyzed. Thirdly, 'Thompson' type models contain a trend variable that is often responsible for most of the 'explanation' of the yield variance. The trend variable was undoubtedly the same for both models thus further complicating the comparison. Their findings could be due to the small change in scale they tested against the proposed scale differences to be expected when small scale physiologically based models are tested. As these models are unlikely to be applied to small area estimates, it would appear to be useful to test the effect of data aggregation on physiologically-based crop yield models.

Limits of Scale interpolation

In an important paper on the limits of predictability of weather, Lorenz concluded that the presence of measurement errors was the final limiter to weather prediction[22]. He found that the rate at which errors double with time places constraints on the final changes in scale. These errors move forward in time and scale until they make forecasts impossible. While not all climatologists and meteorologists share Lorenz's views on the limits of weather predictability, his general methodology must hold true to some extent.

Another limit to consider in reducing scale is the number of increased calculations required. A halving of the distance scale increases the number of computations ten fold. By this process, the capacity of current computers is very quickly reached, not to mention the limit of budgets.

Spectral Gap

Some input variables may not be continuous or defined over the entire range of spatial scales encountered in yield modeling. For example, wind, which is used to model water demand in some yield models, generally receives much less energy from mechanisms at the meso-scale than at the micro-or macro-scale. Because local winds may result from local heating and cooling while large scale winds may result from synoptic scale weather patterns, wind energy 'peaks' both at the local and again at the macro-scale[4]. This lack of 'continuity' of wind and other small scale weather features with increasing spatial scale is called the 'spectral gap' [22].

Barry notes that;

"the frequency spectrum of horizontal wind velocity near the ground shows two distinct peaks of energy contribution, one at the synoptic scale (cyclonic waves) and one in the domain of micro-scale wind eddies. The gaps in the horizontal scale of dimensions between the two classes reflect the uncertain location of the boundary between them." [4]

Priestley referred to this change in the modes of energy and momentum transfers as the 'handover' in scale[28]. Other weather variables may be subject to this same 'spectral gap' when aggregation crosses the boundary between their different modes of transport.

A spectral gap may exist in sensible heat advection. Brakke and others at Nebraska have identified a local and a regional component of sensible heat advection[7]. Local advection arises from local or micro scale changes in surface conditions such as at a irrigated field edge or an abrupt change in vegetation height. Regional advection is due to large scale movement of heated air. In the case of advection, local advection may be an important factor in the locally derived yield equations, but may be insignificant at the regional level, or have 'changed' its character as it moved up in scale.

Priestley also suggested that data may have different forms at different spatial scales. The temperature of a parcel of air changes 'form' as it moves upward from the surface of the earth. At the surface or in the atmospheric boundary layer, air temperature is a result of and 'represents' the local energy fluxes. The temperature of the air near the ground may be largely determined by the balance between the micro-meteorological fluxes of solar radiation, evaporation, and the 'sinks' of these fluxes. As that air moves vertically because of buoyant forces, it quickly becomes removed from its source of heat and begins to cool at a given rate of temperature per increase in height (lapse rate). Thus the temperature for the air mass at a height of 1 KM is very different from the temperature at 1 M height. In like manner, if an air mass is moving horizontally, it will be modified by the surface over which it flows.

Sources of Spatial Error

Errors due to sampling can result from the sampling 'frequency' not being the same as for the underlying phenomena. This mismatching of sampling and data frequencies may result in false interpolated values - 'aliasing errors'. For example, rainfall frequencies may be very short for some periods of the year or in some locations, and much longer in others. Because weather-crop yield models use input variables from many sources, there is no assurance that the variables will all have the same spatial frequency.

The distance between sampling points limits the frequency that can be determined in the weather data. The highest detectable frequency is the 'Nyquist Frequency'. It has a wave length of twice the sampling interval. Higher frequencies may be present but will not be detected. Even Nyquist Frequencies are usually in error, for unless the samples are taken exactly at peaks and troughs, the measured amplitude will be too small [29]. Another error that will possibly be present when spatially discrete data are used to characterize a region is spatial autocorrelation. Spatial autocorrelation is a measure of the degree of interdependence among the sample values (weather, crop observations) between neighboring points. In time series analysis, when error terms between consecutive time periods are related, autocorrelation is said to be present. The same can be true for spatial data points. If measurements at neighboring places are highly correlated, then estimates of model accuracy such as coefficients of determination, will have inflated values. This high spatial autocorrelation can be the result from sampling a large homogeneous region several times, with each sample being treated as an independent observation. Thus while only one observation will be necessary to characterize the area, three or four observations will increase the sample size but not the sample variability. Correlations will thus be artificially high. Testing for spatial autocorrelation will be necessary when yield models are applied to relatively small scales because much inputted data will be interpolated from sparsely spaced measurement points and should therefore be highly related to each other.

Examples of methods of testing for spatial autocorrelation can be found in Cliff and Ord [8]. These are modifications of Markovian equations adapted to

spatial data problems. Another method that can also be used to reduce spatial autocorrelation involves rotating the sampling grid axis to achieve minimum correlation between the sampling axis. Unfortunately, this method requires a regularly spaced grid sample network [21]. This method should be useful if small scale weather data such as county data, are used for yield modeling. In this case, orienting the sampling axis perpendicular to the direction of storm tracks should allow for maximum separation of the weather data.

Synoptic Climatology

Synoptic climatology classifies days based on the similarity of their synoptic-scale weather patterns to other patterns. All the days of a year may thus be classified as belonging to eight or nine different categories of days. Two general methods used to classify weather map patterns appear in the literature. One involves the subjective classification by a trained meteorologist, another uses a map correlation procedure [23]. Orthogonal eigenvectors of weather data can also be used. Methods of classifying weather 'days' have been shown useful in several applications despite its long abused standing with meteorologists as being too 'subjective'. In one comparison of two synoptic classification methods, their utility was assessed and each was found useful[6].

Synoptic classification of weather data seems preferable to the use of static estimates of weather when they will be used in yield models. As an example, regional estimates of solar radiation (R_s), can be estimated in two ways. The historic series of recorded R_s for locations can be tabulated and frequency distributions can be given together with with basic distribution parameters, i.e., means, standard deviations, skewness, and so forth. An alternative approach, using synoptic climatology would be to classify daily R_s by similarity to specific synoptic weather patterns. This method would have the added advantage of being related to the meteorological processes that produced the data as well as being related serially with one another, i.e., one days events would allow a probability statement about the occurrence of the event on the following day. In a synoptic climatological study of S. California, classification by discriminant analysis showed good success in predicting the following 'type' of day 88% of the time[24]. An additional advantage of a synoptic classification approach to estimating and interpolating weather data such as solar radiation (R_s) is that information would be generated concerning the spatial extent to which the data are homogeneous. Within a given 'weather system', the same general energy balance relationships will exist. There are limits to this method, however. As noted by Suckling and Hays,[40], solar radiation can not be estimated from the synoptic type alone. However, as an adjunct to solar radiation measurements, synoptic classification of days offers a useful method for interpolating between existing weather stations. A similar study by the author, relating a simplified synoptic classification to Texas data confirmed their results[38].

An alternative method for estimating both missing data and for computing regional averages of climate data rely on several interpolation techniques. In these methods, missing data are estimated from surrounding data points. Promising methods for such data are forms of Fourier trend surface analysis [3], reciprocal distance weighted methods[43],and orthogonal eigenvector

methods[13]. Comparisons of these methods for wind systems have been made[11]. One possible method can 'weight' neighboring data points for interpolation based on their relative direction[34]. This method should prove useful if knowledge of storm center directions becomes available.

Orthogonal eigenvectors of weather data have been shown to contain very useful mathematical properties in modeling or estimating data. The independence or orthogonality of the combined weather variables is a desirable feature in modeling, but a limitation is their non-physical basis of the derived variables. The principal components are mathematical entities that have no direct meaning in the observable world[13].

Conclusion

In most cases, the scale of application of yield models is constrained by the scale of the final yield statistics. The application of yield models 'below' the county scale is unlikely in any national yield model. However, the choice of the scale of application will in some measure await the development of statistical tools to evaluate the trade-offs between the increased costs of small scale estimates vs. the inaccuracies in applying advanced crop models to very large areas.

As with most fields of inquiry, several approaches to this problem are possible. It is hoped that the increasing division seen between small scale yield modelers and large scale statistical modelers of crop yields will not become hardened into separate camps. The need for a 'merged' approach is vital.

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